# Interfaces: modeling, analysis, numerics

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# Preface

These lecture notes are dedicated to the mathematical modeling, analysis and computation of interfaces and free boundary problems appearing in geometry and in various applications, ranging from crystal growth, tumor growth, biological membranes to porous media, two-phase flows, fluid-structure interactions, and shape optimization. Classical methods from partial differential equations as well as from differential geometry, together with modern methods like the theory of maximal regularity or measure theoretic approaches, now allow for a systematic mathematical theory for interfaces and free boundary problems in many settings. Also, numerical methods based on parametric approaches, level sets or phase fields are now mature enough to deal with interesting phenomena. However, in many applications, quite complex couplings between equations on the interface and equations in the surrounding bulk phases appear, which are still not well understood so far.

We first give an introduction to classical methods from differential geometry and systematically derive the governing equations from physical principles. Then we will analyze parametric approaches to interface evolution problems and derive numerical methods which will be thoroughly analyzed. In addition, implicit descriptions of interfaces such as phase field and level set methods will be analyzed. Finally, we will discuss numerical methods for complex interface evolutions and will focus on two phase flow problems as an important example of such evolutions.

Some parts of the lecture notes have been first used by the first and the third authors in courses they gave to doctoral students of the DFG research training group 2339 *IntComSin: Interfaces, Complex Structures, and Singular Limits in Continuum Mechanics — Analysis and Numerics*. The complete material of this book has been presented at an Oberwolfach seminar in November 2022. We thank the Oberwolfach Research Institute for Mathematics (MFO) for giving us the opportunity to give the lecture series in the Oberwolfach seminar series. We thank all the participants of the seminar for actively taking part in the seminar. Due to their many suggestions for improvements, we were able to substantially enhance the presentation. We would like to thank Paul Hüttl, Jonas Haselböck and Dennis Trautwein for proofreading. We would like especially to thank Eva Rütz and Vera Theus for typesetting parts of the notes and Jiří Minarčík for creating many of the figures in this book.

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