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Z_2 -indices of odd symmetric Fredholm operators

A bounded operator T on a separable, complex Hilbert space is said to be odd symmetric if $I^*T^tI = T$ where I is a real unitary satisfying $I^2 = -1$ and T^t denotes the transpose of T . The Noether index of an odd symmetric Fredholm operator vanishes, but the parity of the dimension of its kernel is shown to be a homotopy invariant that is stable under compact perturbations. The class of real skew-adjoint Fredholm operators for which Atiyah and Singer defined Z_2 -indices is a subset of infinite codimension within the set of odd symmetric Fredholm operators. As example for an odd Z_2 -index theorem, a Z_2 -version of the Gohberg-Krein theorem is presented. An even Z_2 -index theorem leads to a classification of two-dimensional topological insulators with time-reversal symmetry. For these systems, non-trivial Z_2 -invariants imply non-vanishing spin Chern numbers.