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Finite element approximation of large bending isometries

The mathematical description of the elastic deformation of thin plates can be derived by a dimension reduction from three-dimensional elasticity and leads to the minimization of an energy functional that involves the second fundamental form of the deformation and is subject to the constraint that the deformation is an isometry. We discuss two approaches to the discretization of the second order derivatives and the treatment of the isometry constraint. The first one relaxes the second order derivatives via a Reissner-Mindlin approximation and the second one employs discrete Kirchhoff triangles that define a nonconforming second order derivative. In both cases the deformation is decoupled from the deformation gradient and this enables us to employ techniques developed for the approximation of harmonic maps to impose the constraint on the deformation gradient at the nodes of a triangulation. The solution of the nonlinear discrete schemes is done by appropriate gradient flows and we demonstrate their energy decreasing behaviours under mild conditions on step sizes. Numerical experiments show that the proposed schemes provide accurate approximations for large vertical loads as well as compressive boundary conditions.