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Global analysis of the generalised Helfrich flow of curves immersed in $\mathbb{R}^n$

The Helfrich energy is a measure of the elastic bending energy of a manifold, and for surfaces has been in the mind of researchers in one form or another since Poisson's treatise on elasticity in 1812. In 1973 Helfrich used the theory of elastic lipid bilayers to motivate the specific form of the functional which is common today. In its full generality, the functional incorporates an ambient "spontaneous curvature". The presence of even a trivial (constant) spontaneous curvature has historically resisted analysis. In this talk we consider the gradient flow of the functional defined on an immersed curve, with arbitrary codimension. We shall prove that under mild assumptions on the spontaneous curvature the flow exists for all time for initial data with arbitrarily high energy, subconverging to a critical point of a limiting functional. Asymptotic analysis is made particularly difficult by the presence of the spontaneous curvature: we shall present explicit examples where the flow exists for all time but does not converge. Nevertheless (time permitting), following an idea of Ben Andrews, we shall present three conditions under which it is possible to obtain full convergence of the flow. One of these conditions includes the case of the Willmore flow, strengthening a well-known result of Dziuk, Kuwert and Schätzle.