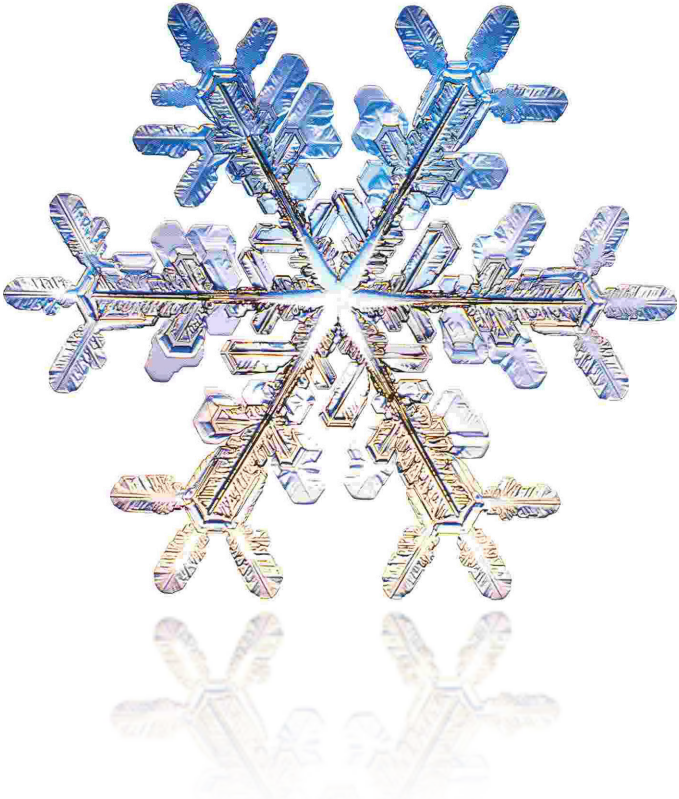
FBP 08

FREE BOUNDARY PROBLEMS, THEORY AND APPLICATIONS



INTERNATIONAL CONFERENCE

9-13 JUNE, 2008

STOCKHOLM, KTH

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Scientific Committee

Ioannis Athanasopoulos, Catherine Bandle, Henri Berestycki, Avner Friedman, Björn Gustafsson, Bernhard Kawohl, Stephan Luckhaus, John Ockendon, Mario Primicerio, Henrik Shahgholian, Nina Uraltseva

Organizing Committee

Teitur Arnarson, Farid Bozorgnia, Anders Edquist, Erik Lindgren, Henrik Shahgholian

Conference Secretary

Ann-Britt Öhman, fbp08@math.kth.se

History

2005 - Coimbra	Portugal
2002 - Trento	Italy
1999 - Chiba	Japan
1997 - Crete	Greece
1995 - Zakopane	Poland
1993 - Toledo	Spain
1990 - Montreal	Canada
1987 - Irsee	Germany
1984 - Maubuisson	France
1981 - Montecatini	Italy

Foreword

FBP08 is the 11th Conference on Free Boundary Problems of the series started in 1981 in Italy and it continuous to be an important event within the FB community.

The Conference has been suggested and supported by the scientific committee of ESF program Global (Global and Geometric Aspects of Nonlinear PDE): www.math.kth.se/global

An EMS mathematical journal, Interfaces and Free Boundaries, www.ems-ph.org/journals/ifb/ifb.php promotes the publication of selected papers in the field.

The electronic newsletter/forum http://fbpnews.org has also served as a communication and information channel for the FB community.

The organizing committee of this program would like to welcome all participants of this meeting to Stockholm. We wish you a pleasant stay and successful meeting.

The Organizing Committee

Schedule SUN-TUE

	SUN JUNE 8	MON JUNE 9	TUE JUNE 10
08:30-09:00		**Registration**	
09:00-09:15		**Openning**	
09:00-10:00		Howison	Fontelos
10:05-10:50		Petrosyan	Vanden Broeck
10:55-11:15		**Coffee**	**Coffee**
11:15-12:00		Rodrigues	Caffarelli
12:05-12:50		Siegel	Roquejoffre
12:50-14:30		**Lunch**	**Lunch**
14:30-16:00 PARALLEL		FS: 1) Shape Opt. 2) Finance	FS: 1) Homogenization 2) Surface waves/ Slamming
16:05-16:30		**Coffee / Poster**	**Coffee / Poster**
16:30-18:00 PARALLEL	Regist-ration	FS: 1) Hele-Show 2) Geometry	FS: 1) Regularity 2) Hyperbolic
18:00-18:15		**Break**	**Break**
18:15-19:30		Short Comm.	Short Comm.

Schedule WED-FRI

WED JUNE 11	THU JUNE 12	FRI JUNE 13	
			08:30-09:00
			09:00-09:15
		Ebert	09:00-10:00
Vázquez	Stefanelli	Burger	10:05-10:50
Coffee	**Coffee**	**Coffee**	10:55-11:15
Hu	Fasano	FS: Plasma	11:15-12:00
Weiss	Dirr		12:05-12:50
Lunch	**Lunch**	**Lunch**	12:50-14:30
Free time	FS: 1) Bio 2) Evolution	FS: New Trends/ Open Problems & **CLOSING**	14:30-16:00 PARALLEL
	Coffee / Poster	**Coffee / Poster**	16:05-16:30
	FS: 1) Food 2) Combustion		16:30-18:00 PARALLEL
Break	**Break**		18:00-18:15
19:00 Official Dinner	Short Comm.		18:15-19:30

Program

1.1 Plenary Lectures

Monday, June 9 – Room: Q1
Session chair: José Rodrigues

⋆ 9:15 - 10:00 **Sam Howison**
From Splish to Splash: Aspects of Water Impact Problems.

⋆ 10:05 - 10:50 **Arshak Petrosyan**
Monotonicity Formulas and the Singular Set in the Thin Obstacle Problem.

Session chair: Catherine Bandle

⋆ 11:15 - 12:00 **José Francisco Rodrigues**
Remarks on a Class of Two Phase Free Boundary Problems

⋆ 12:05 - 12:50 **Michael S. Siegel**
Complex Singularities in Interfacial Fluid Flow and 3D Incompressible Euler Equations.

Tuesday, June 10 – Room: Q1
Session chair: Bernhard Kawohl

⋆ 9:15 - 10:00 **Marco Fontelos**
Free Boundary Problems Involving Electrically Charged Viscous Fluids.

⋆ 10:05 - 10:50 **Jean-Marc Vanden Broeck**
Some New Three Dimensional Free Surface Flows

Session chair: Juan Luis Vázquez

⋆ 11:15 - 12:00 **Luis Caffarelli**
Integral Minimal Surfaces.

⋆ 12:05 - 12:50 **Jean-Michel Roquejoffre**
Free Boundary Problems for the Fractional Laplacian

Wednesday, June 11 – Room: Q1
Session chair: Arshak Petrosyan

⋆ 10:05 - 10:50 **Juan Luis Vázquez**
Porous Medium Flow with Nonlocal Effects

⋆ 11:15 - 12:00 **Bei Hu**
PDE free boundary problems in tumor models

⋆ 12:05 - 12:50 **Georg S. Weiss**
A Nonlinear Frequency Formula and the Singular set of a Free Boundary Problem

Thursday, June 12 – Room: Q1
Session chair: Georg S. Weiss

⋆ 10:05 - 10:50 **Ulisse Stefanelli**
The Energy + Dissipation Functional.

⋆ 11:15 - 12:00 **Antonio Fasano**
Recent Trends in Modelling Cancer Invasion.

⋆ 12:05 - 12:50 **Nicolas Dirr**
Interfaces in Heterogeneous Media

Friday, June 13 – Room: Q1
Session chair: Mario Primicerio

⋆ 9:15 - 10:00 **Ute Ebert**
Streamer Ionization Fronts As a Moving Boundary Problem

⋆ 10:05 - 10:50 **Martin Burger**
Free Boundaries in Biological Aggregation Models.

1.2 Focus Sessions

· Shape optimization
Monday, June 9 – Room: Q1
Organizer: Dorin Bucur

⋆ 14:30 - 15:00 **Antoine Henrot**
The Cylinder Is Not the Optimal Shape for a Pipe

⋆ 15:00 - 15:30 **Edouard Oudet**
Numerical study of an optimal partition problem.

⋆ 15:30 - 16:00 **Dorin Bucur**
Identification of Non Smooth Cracks by Boundary Measurements

· Free boundaries in Finance
Monday, June 9 – Room: Q31
Organizer: Johan Tysk

⋆ 14:30 - 14:50 **John Chadam**
Comparison of Credit Default Models

⋆ 14:50 - 15:10 **Erik Ekström**
Convexity of the Optimal Stopping Boundary for the American Put Option

⋆ 15:15 - 15:35 **Damien Lamberton**
American Options and Parabolic Integro-Differential Operators.

⋆ 15:40 - 16:00 **Goran Peskir**
The British Option

· Hele-Shaw flow & Muskat
Monday, June 9 – Room: Q1
Organizers: Björn Gustafsson
Sam Howison

⋆ 16:30 - 17:00 **Inwon Kim**
The Supercooled Stefan Problem in One Dimension

⋆ 17:00 - 17:30 **Richard Booth**
Miscible Flow through Porous Media

⋆ 17:30 - 18:00 **Anvarbek Meirmanov**
New Free Boundary Problem in Liquid Filtration Arising Via Homogenization

· Geometry of free boundaries
Monday, June 9 – Room: Q31
Organizer: Bernd Kawohl

⋆ 16:30 - 17:00 **Antonio Greco**
On the Convexity of Some Free Boundaries

⋆ 17:00 - 17:30 **Alberto Farina**
A Geometric Approach to Overdetermined Boundary Value Problems

⋆ 17:30 - 18:00 **Cristina Trombetti**
Some Results on a Class of Serrin Type Overdetermined Problems

· Oscillations/homogenization in FBP
Tuesday, June 10 – Room: Q1
Organizer: Ki-Ahm Lee

⋆ 14:30 - 15:00 **Antoine Mellet**
Random Homogenization of Fractional Obstacle Problems

⋆ 15:00 - 15:30 **Tycho Van Noorden**
Crystal Dissolution and Precipitation in Porous Media: Formal Homogenization and Numerical Experiments

⋆ 15:30 - 16:00 **Ki-Ahm Lee**
Viscosity Method in Homogenization

· Surface gravity waves/slamming
Tuesday, June 10 – Room: Q31
Organizers: Jan-Marc Vanden Broeck
John Ockendon

⋆ 14:30 - 14:50 **Alexander Korobkin**
Singular Solutions of Mixed Boundary Value Problems of Water Impact

⋆ 14:50 - 15:10 **Jim Oliver**
Reviving Wagner's theory for high-velocity solid-liquid impact.

⋆ 15:15 - 15:35 **Frédéric Dias**
A Two-Fluid Model for Violent Aerated Flows.

⋆ 15:40 - 16:00 **Yuriy Semenov**
Advanced Technique of Complex Variable for Solving Nonlinear Free Boundary Flows

· Regularity of free boundaries
Tuesday, June 10 – Room: Q1
Organizer: Ioannis Athanasopoulos

⋆ 16:30 - 17:00 **Luis Silvetre**
Regularity Results for the Obstacle Problem for the Fractional Laplacianusing thin Obstacle Problems.

⋆ 17:00 - 17:30 **Norayr Matevosyan**
Regularity of a Free Boundary in Parabolic Problem Without Sign Restriction

⋆ 17:30 - 18:00 **Emmanouile Milakis**
On The Fully Nonlinear Signorini Problem

· Free boundaries in hyperbolic systems
Tuesday, June 10 – Room: Q31
Organizer: Heinrich Freistühler

⋆ 16:30 - 16:50 **Blake Temple**
A Two Parameter Family of Expanding Wave Solutions of the Einstein Equations that Includes the Standard Model of Cosmology.

⋆ 16:50 - 17:10 **Georg Prokert**
Traveling Waves in a Hele-Shaw Type Moving Boundary Problem

⋆ 17:15 - 17:35 **Yuri Trakhinin**
Characteristic Discontinuities in Magnetohydrodynamics

⋆ 17:40 - 18:00 **Robin Young**
Shock-Free Periodic Solutions for the Euler Equations

· Free boundaries in mathematical biology
Thursday, June 12 – Room: Q1
Organizer: Henri Berestycki

⋆ 14:30 - 15:00 **John King**
Multiphase modelling of biological tissue growth.

⋆ 15:00 - 15:30 **Vincent Calvez**
A Chemotaxis Based Model for Concentric Patterning in the Brain.

⋆ 15:30 - 16:00 **Lionel Roques**
Species Persistence: the Optimal Habitat Shape

· Evolution of phase boundaries
Thursday, June 12 – Room: Q31
Organizer: Nicolas Dirr

⋆ 14:30 - 14:50 **Thomas Blesgen**
Modeling the Evolution of Phase Boundaries - a Survey

⋆ 14:50 - 15:10 **Enza Orlandi**
Continuous Interface With Disorder: Even Strong Pinning Is Too Weak in Two Dimensions

⋆ 15:15 - 15:35 **Matthias Röger**
Geometric Methods for the Convergence of Diffuse Interface Models

⋆ 15:40 - 16:00 **Hartmut Schwetlick**
Traveling Wave Solutions for a Non-Convex Fpu Lattice Model Connecting Different Phases

· Food industry
Thursday, June 12 – Room: Q1
Organizer: Mario Primicerio

⋆ 16:30 - 17:00 **Mario Primicerio**
A Non-Standard Free Boundary Problem in Pasta Cooking

⋆ 17:00 - 17:30 **Albergo Mancini**
Modelling Frying Processes

⋆ 17:30 - 18:00 **Andrew Lacey**
Crust Formation in Bread Baking

· Combustion
Thurday, June 12 – Room: Q31
Organizers: Régis Monneau
Henri Berestycki

⋆ 16:30 - 17:00 **Jérôme Coville**
On the Existence of Flame Balls in Lean Media

⋆ 17:00 - 17:30 **Antoine Mellet**
Fronts Propagation in Inhomogeneous Media

⋆ 17:30 - 18:00 **Noemi Wolanski**
Regularity of the Free Boundary for the Limit of an Inhomogenous Singular Perturbation Problem

· Plasma physics
Friday, June 13 – Room: Q1
Organizer: Ute Ebert

⋆ 11:15 - 11:30 **Saleh Tanveer**
Effect of Regularization on Stability of Simple Steadily Propagating Shapes in Streamer Fronts and Other Related Problems.

⋆ 11:30 - 12:00 **Fabian Brau**
Moving Boundary Analysis of Streamer Dynamics

⋆ 12:00 - 12:30 **Darren Crowdy**
Free Boundary Problems with Multiple Interfaces: New Exact Solution

1.3 Contribution Talks

Monday, June 9 – Room: Q1

⋆ 18:15 - 18:30 **Jimmy Lamboley**
Regularity of the Boundary of an Optimal Shape With Convex Constraints

⋆ 18:35 - 18:50 **Sandra Martinez**
A Singular Perturbation Problem for a Quasilinear Operator Satisfying the Natural Growth Condition of Lieberman

⋆ 18:55 - 19:10 **Yuliya V. Namlyeyeva**
Asymptotic Behavior of Solutions to the Dirichlet Eigenvalue Problem in Perforated Domains of General Structure

⋆ 19:15 - 19:30 **Erik Lindgren**
On the two-phase obstacle problem with Hlder continuous coefficients

Monday, June 9 – Room: Q31

⋆ 18:15 - 18:30 **Diogo A. Gomes**
Discrete time, finite state space mean field games

⋆ 18:35 - 18:50 **Kristoffer J. Glover**
Free Boundary Behaviour of Finitely Liquid Markets

⋆ 18:55 - 19:10 **Jacques du Toit**
Selling a Stock at the Maximum

⋆ 19:15 - 19:30 **Teitur Arnarson**
Non-linear PDE methods in terms of stochastics

Tuesday, June 10 – Room: Q1

⋆ 18:15 - 18:30 **Domingo A. Tarzia**
The Coupled Heat and Mass Transfer During the Freezing and Sublimation Processes of High-Water Content Food Materials

⋆ 18:35 - 18:50 **Razvan Gabriel Iagar**
Anomalous Large-Time Behaviour of the P-Laplacian Flow in an Exterior Domain in Low Dimension

⋆ 18:55 - 19:10 **Erwin Vondenhoff**
Long-Time Asymptotics for Some Hele-Shaw Models With Injection or Suction in a Single Point

⋆ 19:15 - 19:30 **Shapour Heidarkhani**
Multiplicity Results to the Existence of Three Solutions for a Class of Neumann Elliptic Systems

Tuesday, June 10 – Room: Q31

⋆ 18:15 - 18:30 **David Siegel**
Behaviour of a Capillary Surface Near a Cusp

⋆ 18:35 - 18:50 **Yasumori Aoki**
An Approximation Technique for the Laplace-Young Capillary Equations in a Circular Cusp Region

⋆ 18:55 - 19:10 **John Andersson**
The Zero Level Set for Certain Weak Solutions with Applications to the Bellman Equations.

⋆ 19:15 - 19:30 **Farid Bozorgnia**
Numerical algorithms for the spatial segregation of competitive systems.

Thursday, June 12 – Room: Q1

⋆ 18:15 - 18:30 **Cristina Caputo**
Motion of Hypersurfaces by the Harmonic Mean Curvature Flow

⋆ 18:35 - 18:50 **Hideki Murakawa**
A Solution of Parabolic Free Boundary Problems by Semilinear Reaction-Diffusion Systems

⋆ 18:55 - 19:10 **Kaj Nyström**
Regularity of Free Boundaries in Two-phase Problems for the p-Laplace Operator

⋆ 19:15 - 19:30 **Silvana de Lillo**
A 2-Phase Free Boundary Problem for a Nonlinear Diffusion-Convection Equation

Thursday, June 12 – Room: Q31

⋆ 18:15 - 18:30 **Matthieu Hillairet**
On Collisions Between Rigid Bodies Inside a Viscous Incompressible Fluid

⋆ 18:35 - 18:50 **Petra Kordulova**
Preisach Model in Hydrology

⋆ 18:55 - 19:10 **Fernando Miranda**
A New Quasi-Variational Inequality for a Maxwell System

⋆ 19:15 - 19:30 **Eugen Varvaruca**
On the Existence of Extreme Waves and the Stokes Conjecture With Vorticity

Abstracts

2.1 Invited Speakers and Focus Sessions

Thomas Blesgen
Max Planck Institute for Mathematics in the Sciences

Modeling the Evolution of Phase Boundaries - a Survey

This talk provides an overview of contemporary mathematical methods to study the evolution of phase boundaries, ranging from classical phase field models to up-to date investigations of the meso- and nano-scales. Various aspects like the effect of stress, nucleation, the role of thermodynamics, simulating the microstructures, recent numerical improvements, and their importance to applications are discussed.

Richard Booth
University of Oxford

Miscible Flow Through Porous Media

Miscible displacement is an enhanced oil recovery technique in which a solvent, miscible with the oil, is injected into an oil reservoir. The flow is usually convection dominated, and when diffusion is neglected we recover the Muskat problem. The solvent is less viscous than the oil it is displacing and so a viscous fingering instability is observed. We study the formation of viscous fingers in a miscible displacement, and show that many thin fingers will form. Numerical simulations show that a mixing zone where the fingers appear. We model the growth of this mixing zone, exploiting the large aspect-ratio of the fingers, to produce a systematic derivation of the Koval model, used in industry. The exact growth rate of the fingers is controlled by behavior near the tips and roots, with further modeling required in this region.

Fabian Brau
CWI Amsterdam

Moving Boundary Analysis of Streamer Dynamics

The minimal PDE model for a streamer discharge consists of two reaction-advection-diffusion equations for electron or ion densities, coupled to the Poisson equation of electrostatics. Ute Ebert showed in her talk how a regularized moving boundary approximation can be formulated, and she presented some solutions of this approximation.

In this talk, we present the extension of linear perturbation theory to arbitrary $\varepsilon > 0$, where ε is the ratio between the regularization length and the radius of the circle. Furthermore, we analyze numerically the nonlinear dynamics of perturbations of the circle solution which is the simplest uniformly translating solution.

We also present a recent study of a periodic array of interacting streamer discharges in a strong homogeneous electric field. Simulations of the minimal PDE model yield a phase diagram with two regions. For large period length and/or large electric field, the streamers branch similarly to single streamers. However, for small period length and field, the streamers reach a uniformly translating motion and do not branch. We also show that the unregularized moving boundary approximation of this dynamics leads to a model identical to the one used to study Hele-Shaw flow in a channel geometry. Analytical uniformly translating solutions for this model are known since the work of Saffman and Taylor. We show that the selected Saffman-Taylor finger fits very well the uniformly translating finger solutions obtained from the simulations of the minimal PDE model, though the regularization is different and the boundary approximation breaks down at the sides of the finger.

Dorin Bucur
Université de Savoie

Identification of Non Smooth Cracks by Boundary Measurements

This talk deals with the identifiability of non-smooth defects (cracks or cavities) by boundary measurements. We prove the uniqueness of the detection by two measurements for arbitrary closed sets satisfying quasi-everywhere a conductivity assumption. This is the case of a large class of closed sets, including sets with infinite number of connected components. This regularity assumption is rather to be related to the Wiener criterion than to the usual boundary smoothness. The weak geometric stability of the detection is proved without any regularity assumption, in the frame of a finite number of defects. It allows to formally prove that a numerical approximation by shape optimization methods converge to the

defect.

Martin Burger
Westfälische Wilhelms-Universität Münster

Free Boundaries in Biological Aggregation Models.

This talk will discuss the appearance and simulation of free boundary motion in biological aggregation models with nonlocal attractive forces. Due to the attractive part clusters will form, which however remain at finite size due to a local repulsive force (nonlinear diffusion). Hence, a similar coarsening behavior as in phase-change models happens, which we analyze and numerically compute based on appropriate gradient flow structures.

Luis Caffarelli
University of Texas at Austin

Integral Minimal Surfaces.

I will talk about Integral Minimal Surfaces We will discuss movement by "integral curvature" and integral minimal surfaces. This type of problems occur when long range interactions determine the local diffusion speed along a surface.

Vincent Calvez
École Normale Supérieure

A Chemotaxis Based Model for Concentric Patterning in the Brain.

The Keller-Segel model for cell-to-cell attraction is able to reproduce the phenomenon of critical mass: cells spatially organize themselves only if the total amount of cells is sufficiently high in some sense. A typical example is the cell density's blow-up occurring when the total mass is above a certain threshold in 2D.

In this talk we briefly review the Keller-Segel model and its recent extensions. As an application we suggest a 3-species model that describes pattern formation arising in a variant of Multiple Sclerosis. Instead of being homogeneous plaques, the areas of damaged myeline are in fact concentric rings in Baló's Concentric Sclerosis. We show how this simple 3-species model based on chemotaxis describes correctly the dynamics of this disease, and we derive several qualitative properties such as an interesting correlation between pattern formation and aggressivity of the disease.

This is joint work with H.R. Khonsari. It is closely related to recent investigations by Nadin, Perthame and Ryzhik.

John Chadam
University of Pittsburgh

Comparison of Credit Default Models

Similarities and differences in the structural (value-of-firm), copula and intensity modeling of credit default will be examined. A complete description of the relationship of the default probability of a single firm and its default barrier will be given. We will then outline how default correlation between two firms is naturally included in these structural models and derive the joint default probability of the firms in this context. Comparisons will be made with the joint default probabilities obtained from copula and intensity models in the context of tail dependence (independence of extreme events), calibration to first-to-default credit default swaps (FtD CDS) and the distribution of time between defaults. This is joint work with students Junming Huang, Bo Shi and Lung Kwan Tsui.

Jérôme Coville
Max Planck Institute for Mathematics in the Sciences

On the Existence of Flame Balls in Lean Media

I will present new existence results for solution of a reaction diffusion model of spherical flame in lean media. I will first focus on a simplified version of the model where the Arrhenius law is replaced by an Heavyside function with an ignition temperature and heat losses due to radiation are modelled by a linear function

of the temperature. I will present an explicit construction of the solutions and a full description of the set of solutions. Then, I will present some existence results concerning the full reaction diffusion system.

Darren Crowdy
Imperial College, London

Free Boundary Problems With Multiple Interfaces: New Exact Solutions

This talk will present new analytical solutions to a free boundary problem in which multiple interacting interfaces are present. The mathematical problem appears in a wide variety of physical problems (including Hele-Shaw flows and "streamers" in electric fields). The new results are derived using some analytical techniques for multiply connected planar geometries recently developed by the speaker and are related to prior work by Tanveer [Phys. Fluids, vol 30, (1987)] on steady bubbles in Hele-Shaw channels.

Frédéric Dias
Ecole Normal Supérieure, Cachan

A Two-Fluid Model for Violent Aerated Flows.

In the study of ocean wave impact on structures, one often uses Froude scaling since the dominant force is gravity. However the presence of trapped or entrained air in the water can significantly modify wave impacts. When air is entrained in water in the form of small bubbles, the acoustic properties in the water change dramatically and for example the speed of sound in the mixture is much smaller than in pure water, and even smaller than in pure air. While some work has been done to study small-amplitude disturbances in such mixtures, little work has been done on large disturbances in air-water mixtures. We propose a basic two-fluid model in which both fluids share the same velocities. It is shown that this model can successfully mimic water wave impacts on coastal structures. Even though this is a model without interface, waves can occur. Their dispersion relation is discussed and the formal limit of pure phases (interfacial waves) is considered. The governing equations are discretized by a second-order finite volume method.

Numerical results are presented. It is shown that this basic model can be used to study violent aerated flows, especially by providing fast qualitative estimates.

Nicolas Dirr
University of Bath

Interfaces in Heterogeneous Media

We consider energies which consist of an interfacial energy and a heterogeneous (periodic or random) "bulk"or "volume" term, with heterogeneities that change on a "small" scale. This competition of the interfacial energy and the heterogenities creates a complex energy landscape. We present some results for the qualitative behavior on large scales, both for the energies and for the associated gradient flows.

Ute Ebert
CWI Amsterdam and Eindhoven Univ. Techn.

Streamer Ionization Fronts As a Moving Boundary Problem .

Streamer discharges are the earliest stages of sparks and lightning; they are growing cold plasma channels that propagate due to the nonlinear coupling of electron drift, impact ionization reaction and space charge effects, which leads to a strong field enhancement at the propagating tips of the streamer fingers. Tens of kilometers large sprite discharges above thunderclouds are driven by the same physical mechanisms. As streamers and sprites propagate with velocities of 10^5 to 10^7 m/s, experiments and observations only recently allow to characterize them properly.

The minimal model for a streamer discharge (e.g., in pure gases like nitrogen or argon) consists of two reaction-advection-diffusion equations for electron or ion densities, coupled to the Poisson equation of electrostatics. Numerical solutions show the formation of a thin space charge layer around the ionized streamer interior. A moving boundary approximation for this thin layer is of viscous fingering type. As a regularization of the boundary dynamics, we have suggested and approximately derived a mixed Dirichlet-Neumann boundary condition, similar to kinetic undercooling.

We have thoroughly analyzed the stability of convecting circles in this boundary problem, as these are the simplest uniformly translating solution. For a particular ratio ε of regularization length over radius, the linear perturbation theory about the convecting circle can be solved exactly, and universal asymptotes for time to ∞ as well as the discrete spectrum can be determined. The extension of linear perturbation theory to arbitrary $\varepsilon > 0$, nonlinear solutions as well as the dynamics of finger solutions will be treated by Fabian Brau in his talk.

The moving boundary analysis is joint work with Fabian Brau, Chiu-Yen Kao, Bernard Meulenbroek, Lothar Schäfer and Saleh Tanveer.

Erik Ekström
Uppsala University

Convexity of the Optimal Stopping Boundary for the American Put Option

We show that the optimal stopping boundary for the American put option is convex in the standard Black-Scholes model. The methods are adapted from ice-melting problems and rely upon studying the behavior of level curves of solutions to certain parabolic differential equations.

Alberto Farina
Université de Picardie Jules Verne

A Geometric Approach to Overdetermined Boundary Value Problems

We use the theory of isoparametric surfaces to prove the following symmetry result:
if the overdetermined elliptic (not necessarily uniformly elliptic) boundary value problem:

$$\begin{cases} -div(A|\nabla u|\nabla u) = 1 & \text{in } \Omega \\ u = 0,\ |\nabla u| = c & \text{on } \partial\Omega \end{cases}$$

admits a weak $C_0^1(\bar{\Omega})$-solution in a connected bounded open set $\Omega \subset \mathbb{R}^N$, then Ω must be an euclidean ball.

The above result is proved under *minimal assumptions* on both Ω and the function A (this is a joint work with B. Kawohl).

Antonio Fasano
Università degli Studi di Firenze

Recent Trends in Modelling Cancer Invasion.

The literature on cancer modelling has increased at an impressive pace during the last decade, addressing a number of different topics. Several mathematical models have been formulated for tumors with a prescribed geometry (namely the multicellular spheroids and the so-called tumor cords), dealing with various aspects of their evolution (growth, metabolism, mechanical behavior, angiogenesis, treatments etc.), both in vitro and in vivo. The possibility that such simple shapes can be unstable, giving rise to more complicated structure (even of fractal type) has also been analyzed in a series of papers. Recently much attention has been devoted to the critical problem of tumor invasiveness, which in turn involves the study of cells motility (chemotaxis, haptotaxis, intra- and extra-vasation), as well as the ability of tumor cells to exert an aggressive action on the host tissue. All these topics seem to be relevant to the phenomenon of metastasis. Here we want to concentrate on the question of the acid-mediated tumor invasion. Such a process is based on the fact that the host tissue is more sensitive to pH lowering than the tumor cells. Thus the tumor invasion can be favored by the production of an acidic environment. The latter is a consequence of the products of a metabolism shifted towards glycolysis, which is typical of hypoxic tumors. Two classes of papers will be considered. The first class studies the progression of the tumor by assuming some simple law of production of an excess of H+ ions and contains papers dealing with one-dimensional traveling waves or with spherical tumors (vascular or not). The second class is concerned more specifically with the metabolism of spheroids responsible for the pH lowering. The two classes present interesting features and are in a sense complementary, suggesting that much progress can result from combining the two approaches.

Marco Fontelos
Consejo Superior de Investigaciones Científicas

Free Boundary Problems Involving Electrically Charged Viscous Fluids.

Free boundary problems involving electrically charged fluids and the presence of surface tension forces are attracting the attention of broad communities of physicists and engineers. The main reason is the possibility of controlling the behaviour of fluids at micro and nanometer length scales by means of elec- tric and magnetic fields with all the potential applications that should provide. Mathematically, these problems involve solving Stokes equations in the fluid domain sub ject to boundary conditions that balance viscous stresses with sur- face tension forces and electrostatic repulsion of charges. If the fluid is partly in contact with a solid, then no-slip boundary conditions are imposed at the solid-fluid interface. The presence of repulsive, and hence destabilizing, forces at the interface that oppose the stabilizing surface tension forces gives rise to various interesting phenomena. These include: 1) the existence of nonspherical equilibrium con- figurations for levitating drops that can be characterized as symmetry-breaking bifurcations of spherical configurations, 2) the existence of instabilities of the interface leading to the formation of finite-time geometrical singularities in the form of cones and emission of jets, 3) the phenomenon of electrowetting, con- sisting in the control of the wetting properties of fluids by means of electric fields. Mathematically, problem 1) is addressed via Crandall-Rabinowitzs theorems suitably adapted to the study of free boundary problems, problem 2) is studied via boundary integral formulations and local analysis near the singularities and problem 3) is essentially variational. We will review recent contributions as well as some open problems.

Antonio Greco
University of Cagliari

On the Convexity of Some Free Boundaries

This talk deals with the free boundary arising in some variational problems, with or without obstacle, including the well-known *dam problem*. The free boundary is the set $\{ x : u(x) = 0 \}$, where u minimizes a convenient convex functional in some convex space of admissible functions. In a work in progress with B. Kawohl, u is compared via maximum principle with its quasi-concave envelope u^* (the smallest quasi-concave function above u). It turns out that $u = u^*$ and therefore every level set of u is convex. A similar approach was formerly used by Colesanti and Salani (2003) and by Cuoghi and Salani (2006) in the case when

the set $\{u = 0\}$ is prescribed.

Antoine Henrot
Université Henri Poincaré

The Cylinder Is Not the Optimal Shape for a Pipe

In this talk, we are interested in the optimal shape of a pipe (inlet and outlet are fixed and the volume is prescribed). We consider an incompressible fluid, subject to the Navier-Stokes equations with classical boundary conditions on the boundary of the cylinder (velocity profile given on the inlet, no slip condition on the lateral boundary and outlet-pressure condition on the outlet). We are interested in the following question: is the cylinder the optimal shape for the criterion "energy dissipated by the fluid"? We prove that it is not the case. For that purpose, we explicit the first order optimality condition, thanks to adjoint state and we prove that it is impossible that the adjoint state be a solution of this over-determined system (joint work with Yannick Privat).

Sam Howison
University of Oxford

From Splish to Splash: Aspects of Water Impact Problems

I shall describe asymptotic approaches to a range of 'violent impact' problems involving collisions between liquid and solid or liquid bodies. Applications include impact of a blunt body on water, early stages of droplet collison, and high Froude number shallow-water flows on a sloping base. Mathematical connections range from variational inequalities to delta shocks.

Bei Hu
University of Notre Dame

PDE Free Boundary Problems in Tumor Models

We shall discuss the recent progress (joint work with Avner Friedman) on the PDE tumor models, the linear stability of the tumor, the nonlinear stability of the tumor, and the comparison of the stability between different models. These comparisons have implications on the physical problems.

Inwon Kim
UCLA

The Supercooled Stefan Problem in One Dimension

We investigate the supercooled Stefan problem

$$\begin{cases} \rho_t - \rho_{xx} = 0 & \text{in } \{\rho > 0\} \\ V_{x,t} = -|\rho_x| & \text{on } \partial\{\rho > 0\} \end{cases}$$

where $V_{x,t}$ denotes the outward velocity of the free boundary $\partial\{\rho > 0\}$ with respect to the positive phase. We will discuss corresponding discrete model using interacting particles, corresponding conservation laws and properties of the weak solution depending on the size of the initial data. This is joint work with Lincoln Chayes.

John King
University of Nottingham

Multiphase modelling of biological tissue growth.

Tissue engineering raises some novel modelling challenges and some continuum models developed to describe aspects of the growth process will be described. Their relationship to free-boundary-problem and nonlinear-PDE formulations that arise in other contexts will be mentioned.

Alexander Korobkin
University of East Anglia

Singular Solutions of Mixed Boundary Value Problems of Water Impact

Initial stage of plate impact onto a liquid free surface is considered in two-dimensional and axisymmetric configurations. The ratio of the plate displacement to the plate dimension plays the role of a small parameter. Asymptotic analysis is performed within the ideal and incompressible liquid model. Method of matched asymptotic expansions is used to derive the second order outer solution, which is valid outside small vicinities of the plate edge, and the equations which govern the flow close to the plate edge. It is shown that the initial flow close to the plate edge is two-dimensional and self-similar in the leading order. This flow is governed by non-linear boundary-value problem with unknown shape of the free surface. The inner velocity potential was calculated numerically by boundary-element method. The far-field asymptotics of the inner solution is analytically derived. This asymptotics reveals an eigen solution which gives rise to important singular contribution to the second-order outer velocity potential. In addition to this singular eigen solution the second-order velocity potential is singular on its own at the plate edge. The singular solution of the outer second-order problem is obtained with the help of theory of analytic functions in two-dimensional case and a combination of special functions in the axisymmetric case. The second-order outer solution is matched with the leading order inner solution. Uniformly valid distribution of the hydrodynamic pressure over the plate is obtained. Initial asymptotic behavior of the total hydrodynamic force acting on the entering plate is analytically evaluated and compared with the results of numerical calculations by non-linear potential solver. It is shown that the eigen solution of the outer problem gives the most important contribution to the total force.

The eigen solutions of the plate impact problem are used to derive the second-order velocity potential and the pressure distribution in the problem of wave impact onto an infinite rigid plate. In the latter problem the wetted area of the plate grows in time and the leading order inner solution is known in analytical form.

Joint work with Iafrati Alessandro

Andrew Lacey
Heriot-Watt University

Crust Formation in Bread Baking

A model for the baking of bread, and in particular the formation of a crust during baking, is presented. The crust is the outermost part of a bread loaf where the final density is significantly higher than in the crumb, the main part of the

loaf. Our model is based on a collapse mechanism, whereby raised pressures in the interior of the loaf, due to thermal expansion and water evaporation, squash bubbles in the outer part of a bread loaf at the same time as the bread sets and fractures. The latter process allows vapour to escape from bubbles which can then shrink. The bubble collapse occurs at a free boundary which moves into the bread loaf. A second free boundary models the boiling off of water.

Damien Lamberton
Université Paris-Est Marne-la-Vallée

American Options and Parabolic Integro-Differential Operators

The price of an American option is generally computed by solving an optimal stopping problem. In this talk, based on joint work with Mohammed Mikou, we will first discuss the connection between optimal stopping of Lévy processes and parabolic integro-differential inequalities. We will then focus on some qualitative properties of the American put price in exponential Lévy models. In particular, we will study the associated free boundary, which is related to the optimal exercise strategy. We will also investigate the so-called smooth fit property in this setting.

Ki-Ahm Lee
Seoul National University

Viscosity Method in Homogenization

In this talk we are going to consider viscosity method in the homogenizations of the highly oscillating elliptic or parabolic obstacle problems and of some nonlinear problems in perforated domain. This viscosity method can be applicable for the fully nonlinear equations. For the simplicity, we may consider obstacles that are consisted of cylindrical columns distributed periodically. If the decay rate of the capacity of columns is too high or too small, the limit of u_ε ends up with trivial solutions. The critical decay rates of having nontrivial solution are obtained with construction of barriers. And discrete gradient estimate and almost flatness in each cell will give us a concept of convergence. We also show the limit of u_ε satisfies a homogenized equation with a term showing the effect of the highly oscillating obstacles in viscosity sense. We will also discuss the other

issues in different nonlinear problems.

Alberto Mancini

A Model for the Deep-Frying of an Undeformable Potato-Slice Is Discussed.

The derivation of the model from the basic principles of mass and enthalpy conservation is presented. Attention is focused on the early stage of vaporization, for which an explicit solution is provided, thus bypassing the computational difficulties connected with the appearance of an interface. We present numerical simulations that show a satisfactory agreement with the available experimental data. Joint work with A. Fasano.

Norayr Matevosyan
University of Vienna

Regularity of a Free Boundary in Parabolic Problem Without Sign Restriction

We consider a parabolic obstacle-type problem without sign restriction on the solution, for which we obtain the exact representation of the global solutions (i.e., solutions in the entire half-space $\{(x,t) \in \mathbb{R}^{n+1} : x_1 > 0\}$) and study the local properties of the free boundary near a fixed one. We also prove the smoothness of the free boundary under a homogeneous Dirichlet condition on the given boundary. This is a joint work with D. Apushkinskaya and N.N. Uraltseva.

Anvarbek Meirmanov
Belgorod State University, Russia, TU Minich, Germany **New Free**

Boundary Problems in Liquid Filtration Arising Via Homogenization

In the present talk we consider new free boundary problems describing a joint filtration of two immiscible incompressible viscous fluids with different viscosities and densities. On the micro-level the mathematical model consists of Stokes equation for the liquid velocity in the pore space, Lame equation for the displacements of the elastic solid matrix, continuity conditions on the joint boundary "liquid-solid" and transport equations for the unknown density and viscosity of the liquid. This problem is very hard to tackle due to its nonlinearity and the fact that its main differential equations involve non-smooth oscillatory coefficients, both big and small, under the differentiation operators. To simplify the model we suggest a homogenization procedure when the dimensionless size ε of pores tends to zero, while the porous body is geometrically periodic. For the single fluid the structure of homogenized equations depend on the limits:

$$\lim_{\varepsilon \searrow 0} \alpha_\lambda(\varepsilon) = \lambda_0,\ \lim_{\varepsilon \searrow 0} \alpha_\mu(\varepsilon) = \mu_0,\ \lim_{\varepsilon \searrow 0} \frac{\alpha_\mu}{\varepsilon^2} = \mu_1,$$
$$\alpha_\mu = \frac{2\mu}{\tau L g \rho_0},\ \alpha_\lambda = \frac{2\lambda}{L g \rho_0},$$

where μ is the viscosity of fluid, λ is the elastic Lamé constant, L is a characteristic size of the domain in consideration, τ is a characteristic time of the process, ρ_0 is the mean density of water, and g is the value of acceleration of gravity. There are two scenarios to get new homogenized free boundary problems. The first scenario proposes a limiting procedure as $\varepsilon \searrow 0$ for free boundary problems on the micro level. As a result we obtain a new model together with its solvability. According to the second scenario we first fulfill the passage to the limit as $\varepsilon \searrow 0$ for each domain, occupied by single liquid, and then add the transport equations for the density and viscosity of the liquid to already obtained homogenized equations, describing the motion of the medium. Both ways give the same mathematical models, but for the second scenario the correctness of the obtaned model remains open. Namely, if

1) $\mu_0 = 0$, $0 < \mu_1 < \infty$ and $\lambda_0 = \infty$ we arrive at the well – known Muskat problem, which is still unsolved. If

2)$\mu_0 = 0$, $\mu_1 < \infty$ and $0 < \lambda_0 < \infty$, then we arrive at new Terzaghi-Biot-Muskat problem, which consist of the Terzaghi-Biot system for the filtration of a viscous liquid in the elastic solid skeleton coupled with the transport equations for the density and viscosity of the liquid. Finally, if

3) $0 < \mu_0, \lambda_0 < \infty$ then the limiting regime is described by the system of visco-elasticity for the displacements of the medium coupled with the corresponding transport equations for the density and viscosity of the liquid. For this last problem we prove the existence and uniqueness of the generalized solution.

Antoine Mellet
The University of British Columbia

Random Homogenization of Fractional Obstacle Problems

We study the homogenization of fractional obstacle problems in a perforated domain, when the holes are periodically distributed and have random shape and size. The main assumption is that the "size" of the holes is stationary ergodic. Fractional obstacle problems arise, for instance, when modeling molecules transport across semi-permeable membranes.

Antoine Mellet
The University of British Columbia

Fronts Propagation in Inhomogeneous Media

I'll review various results concerning combustion fronts propagation in periodic and non periodic media.

Emmanouile Milakis
University of Washington

On The Fully Nonlinear Signorini Problem

We will discuss recent results on free boundary problems for fully nonlinear equations which are motivated by applications in elasticity, such as the fully nonlinear Signorini problem. We will concentrate on the tools which will lead us to regularity of solutions for those free boundary problems, in which the free interface has an active role in the overall process. This is a joint work with L. Silvestre.

Mario Primicerio
U.Dini

A Non-Standard Free Boundary Problem in Pasta Cooking

The talk will describe a work in progress in which a free boundary problem for the diffusion equation is studied modeling the cooking of pasta. Water infiltration and gelatinization seem to be the most relevant processes occurring during the cooking; their interplay originates a problem which is non-standard and presents some interest in itself.

Jim Oliver
OCIAM, Oxford

Reviving Wagner's Theory for High-Velocity Solid-Liquid Impact

this is the text of abstract.

Enza Orlandi
Universitá di Roma 3

Continuous Interface With Disorder: Even Strong Pinning Is Too Weak in Two Dimensions

I present recent results about a statistical mechanics model of continuos height effective interfaces in the presence of a delta-pinning.

Edouard Oudet
University of Savoie

Numerical study of an optimal partition problem.

We introduce a new numerical approach in order to approximate the partition which minimizes the sum of its fundamental modes.

Goran Peskir
University of Manchester

The British Option

We present a new put/call option where the buyer may exercise at any time prior to maturity whereupon his payoff is the 'best prediction' of the European payoff under the hypothesis that the true drift of the stock price equals a contract drift. Inherent in this is the protection feature which is key to the British option. Should the option holder believe the true drift of the stock price to be unfavorable (based upon the observed price movements), he can substitute the true drift with the contract drift and minimize his losses. With the contract drift properly selected the British put option becomes a more 'buyer friendly' alternative to the American put: when stock price movements are favorable, the buyer may exercise rationally to very comparable gains; when price movements are unfavorable he is afforded the unique protection described above. Moreover, the British put option is always cheaper than the American put. In the final part we present a brief review of optimal prediction problems which preceded the development of the British option.

This is a joint work with F. Samee (Manchester).

Arshak Petrosyan
Purdue University

Monotonicity Formulas and the Singular Set in the Thin Obstacle Problem.

We construct two new one-parameter families of monotonicity formulas to study the free boundary points in the lower dimensional obstacle problem. The first one is a family of Weiss type formulas geared for points of any given homogeneity and the second one is a family of Monneau type formulas suited for the study of singular points. We show the uniqueness and continuous dependence of the blowups at singular points of given homogeneity. This allows to prove a structural theorem for the singular set.

Our approach works both for zero and smooth non-zero lower dimensional obstacles. The study in the latter case is based on a generalization of Almgren's frequency formula, first established by Caffarelli, Salsa, and Silvestre.

This is a joint work with Nicola Garofalo.

Georg Prokert
TU Eindhoven

Traveling Waves in a Hele-Shaw Type Moving Boundary Problem

We discuss a 2D moving-boundary problem for the Laplacian with Robin boundary conditions in an exterior domain. It arises as model for Hele-Shaw flow of a bubble with kinetic undercooling regularization and is also discussed in the context of models for electrical streamer discharges.

The corresponding evolution equation is given by a degenerate, nonlinear transport problem with nonlocal lower-order dependence. We identify the local structure of the set of traveling-wave solutions in the vicinity of trivial (circular) ones. We find that there is a unique nontrivial traveling wave for each velocity near the trivial one. Therefore, the trivial solutions are unstable in a comoving frame.

The degeneracy of our problem is reflected in a loss of regularity in the estimates for the linearization. Moreover, there is an upper bound for the regularity of its solutions.

(Joint work with M. Günther, Universität Leipzig)

José Francisco Rodrigues
University of Lisbon/CMAF

Remarks on a Class of Two Phases Free Boundary Problems

We present some properties of the solution to a class of free boundary problems with two phases. Under a natural nondegeneracy assumption on the interface, for which a sufficient condition is given, we prove a continuous dependence result for the characteristic functions of each phase and we establish sharp estimates on the variation of its Lebesgues measure with respect to the L1-variation of the data, in a rather general framework.

Jean-Michel Roquejoffre
Institut de Mathématiques, Université Paul Sabatier

Free Boundary Problems for the Fractional Laplacian

We discuss here local properties - optimal regularity, nondegeneracy, smoothness - of a free boundary problem involving the fractional Laplacian, generalising the classical phase transition problem for the standard Laplacian with gradient jump. Our equations are relevant models for boundary reactions, but also to reaction-diffusion processes involving non-Gaussian diffusion.
The nonlocality of the fractional laplacian renders the problem nontrivial, and the key tool is the Caffarelli-Silvestre extension formula, which transforms the model into a codimension 2 free boundary problem.
Joint work with L. Caffarelli and Y. Sire.

Lionel Roques
INRA Avignon

Species Persistence: the Optimal Habitat Shape

In a binary environment made of habitat and non-habitat regions, we study species persistence through a reaction-diffusion model. Species survival both depends on habitat abundance and shape. For a fixed habitat abundance, we describe the shapes that maximize the chances of survival, in bounded and infinite-periodic environments.

Matthias Röger
Max Planck Institute for Mathematics in the Sciences

Geometric Methods for the Convergence of Diffuse Interface Models

Phase field models are a common approach to phase separation processes. They are often given as gradient flows of a diffuse surface energy that goes back to van der Waals and Cahn-Hilliard. Often these models correspond formally to a sharp interface limit. To justify the passage to the limit techniques from geometric measure theory have proved useful. We report on progress and limitations of this approach.

Hartmut Schwetlick
University of Bath

Traveling Wave Solutions for a Non-Convex Fpu Lattice Model Connecting Different Phases

The existence of travelling waves in atomistic models for martensitic phase transitions is studied. The elastic energy is assumed to be piecewise quadratic, with two wells representing two stable phases. The focus is on subsonic waves with strains exploring both wells of the energy. We prove rigorously the existence of travelling waves in a one-dimensional chain of atoms. The wave is 'heteroclinic' in the sense that asymptotically the strains are contained in different wells of the energy.

Yuriy Semenov
National Academy of Sciences of Ukraine

Advanced Technique of Complex Variable for Solving Nonlinear Free Boundary Flows

Historically, a progress in solving problems of two-dimensional free boundary potential flows is based on a development of the theory of complex variable. Since any analytical function meets the requirements of a fluid incompressibility and zero vorticity, the problem is to find such analytical function which satisfies to given boundary conditions. The present talk will be focused on determination of a complex function from its modulus and argument or its real part and argument, which are given on the boundary of a simply connected domain. In combination with Chaplygins singular point method it makes possible to determine expressions for a complex velocity and for a derivative of the complex potential of an arbitrary unsteady free boundary flow. These expressions contain in an explicit form the modulus and argument of the velocity defined as functions of a parameter variable and time. The dynamic and kinematic boundary conditions lead to a system of integral and integro-differential equations for determination of these unknown functions. The proposed method recently been applied to the solution of self-similar water entry problems and time-dependent problems of Hele-Shaw flows with and without surface tension.

Michael S. Siegel
New Jersey Institute of Technology

Complex Singularities in Interfacial Fluid Flow and 3D Incompressible Euler Equations.

One of the most interesting phenomena in free surface flows is singularity formation on the interface. Examples include topological singularities, such as pinch-off of a liquid thread, or the formation of a curvature singularity on an evolving vortex sheet. In the first part of this talk, we describe how singularity formation can be understood by analytically extending the variable which parameterizes the interface and analyzing the singularity motion in the complex plane. As an example, we discuss the recent construction of singular solutions for two-phase flow in porous medium or Hele-Shaw cell. In the second part, we describe a new approach for the construction of complex singular solutions to the 3D Euler equations that is motivated by the results for interfaces.

Luis Silvestre
Courant Institute of Mathematical Sciences

Regularity Results for the Obstacle Problem for the Fractional Laplacianusing Thin Obstacle Problems

We obtain the optimal regularity of the solutions and the regularity of the free boundary of the obstacle problem for the fractional Laplacian. We rewrite the original nonlocal problem as an equivalent local problem in one dimension more. The obstacle problem for the fractional Laplacian turns out to be equivalent to an appropriate "thin" obstacle problem. In this way we can study a problem involving the fractional Laplacian using standard PDE (local) tools. This is a joint work with Luis Caffarelli and Sandro Salsa.

Ulisse Stefanelli
IMATI, Pavia

The Energy + Dissipation Functional.

The evolution of dissipative systems is governed by the interplay between energy and dissipation. Roughly speaking, evolution is driven by energy whereas dissipation sets the reference metric landscape (in other words, energy decreases along dissipation geodesics). Although energy and dissipation play such substantially different roles in the evolution, they get summed together and minimized in the standard implicit Euler time-discrete scheme.

The purpose of this talk is to report on a recent progress in the variational treatment of the dissipative evolution which basically consists in a time-continuous version of this fact. In particular, I will focus on a class of global-in-time functionals recently proposed by Mielke & Ortiz where the (weighed) sum energy+dissipation comes directly into play. The interest in this perspective is that of possibly applying the tools of the Calculus of Variations (e.g., Γ-convergence, relaxation, approximation) to evolution. I will review some flow and the rate- independent case.

This is a joint project with A. Melike (Berlin).

Saleh Tanveer
The Ohio State University

Effect of Regularization on Stability of Simple Steadily Propagating Shapes in Streamer Fronts and Other Related Problems

An important problem in pattern formation is the stability of propagating shapes. In the context of streamers, modeled as a sharp front where equations similar to that of quasi-steady crystal growth with kinetic regularization arise, previous work by Brau et al show steadily propagating circular shapes are linearly stable at a particular value of kinetic regularization. Recent work with collaborators will be presented on the spectral analysis of the linear stability problem for arbitrary value of the regularization parameter. It is shown that all eigenvalues are negative. We compare and contrast these results with earlier work on the mathematically related viscous fingering problem with surface tension for circular and more general shapes. In the viscous fingering case, we also present some rigorous mathematical results on the nonlinear stability problem.

Blake Temple

UC Davis

A Two Parameter Family of Expanding Wave Solutions of the Einstein Equations that Includes the Standard Model of Cosmology.

In this talk I discuss recent joint work with Joel Smoller in which we derive an exact two parameter family of expanding wave solutions of the Einstein equations that includes the critical (flat space $k = 0$) Friedmann universe in the standard model of cosmology. All of the spacetime metrics associated with this family apply when the equation of state is given by $p = c^2/3\ \rho$, correct for early Big Bang physics, after inflation. By expanding solutions near the standard model, about the center, to leading order in the Hubble length, we find a one-parameter family of expanding spacetimes that represent a perturbation of the standard model. We then show that there exists a coordinate system in which the perturbed spacetimes agree with the standard model in each space-like time-slice, (a flat metric with a time dependent scale factor), but there are small corrections to the Hubble constant, the particle velocities, and there is a small space-time cross term. Since exact non-interacting expansion waves represent possible time-asymptotic wave patterns for conservation laws, we wonder whether it is possible that these corrections to the standard model might account for the anomalous acceleration of the galaxies, without the introduction of the cosmological constant. (Articles and commentaries can be found on author's website:
http://www.math.ucdavis.edu/ temple/articles/) .

Yuri Trakhinin

Sobolev Institute, Novosibirsk

Characteristic Discontinuities in Magnetohydrodynamics

In ideal compressible magnetohydrodynamics (MHD) there are three types of characteristic discontinuities: current-vortex sheets, contact discontinuities, and Alfven discontinuities. We survey recent results in the study of existence and stability of such characteristic free boundaries for the MHD equations. The main attention is devoted to current-vortex sheets [1, 2] but, for example, stability results for Alfven discontinuities [3] are also discussed.

The main result for current-vortex sheets is a local-in-time existence theorem [2]. Namely, we prove the local-in-time existence of solutions with a surface of current-vortex sheet of the MHD equations in three space dimensions provided

that a stability condition [1] is satisfied at each point of the initial discontinuity. The fact that the Kreiss-Lopatinski condition is satisfied only in a weak sense yields losses of derivatives in a priori estimates. Therefore, we prove our existence theorem by a suitable Nash-Moser-type iteration scheme.

At last, in the talk we also concerned with incompressible current-vortex sheets [4, 5]. In spite of the fact that the equations of incompressible MHD are not hyperbolic, for the linearized free boundary value problem for planar incompressible current-vortex sheets we use a technique that is like Kreiss' symmetrizers technique. Namely, we derive an a priori estimate in the stability domain found a long time ago by Syrovatskij, and the crucial point in deriving this estimate is the construction of a symbolic symmetrizer for a nonstandard elliptic problem for the small perturbation of total pressure [5]. This symmetrizer is a kind of "elliptically degenerate Kreiss' symmetrizer."

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Cristina Trombetti
Università degli Studi di Napoli "Federico II"

Some Results on a Class of Serrin Type Overdetermined Problems

We extend the classical symmetry result of Serrin to the case of Hessian equations presenting an alternative proof which does not rely explicitly on the maximum principle. The techniques used can be also adapted to obtain stability results with respect to perturbations of boundary data.

Tycho Van Noorden
Technische Universiteit Eindhoven

Crystal Dissolution and Precipitation in Porous Media: Formal Homogenization and Numerical Experiments

We investigate a two-dimensional micro-scale model for crystal dissolution and precipitation in a porous medium. The model contains a free boundary and allows for changes in the pore volume. Using a level-set formulation of the free boundary, we apply a formal homogenization procedure to obtain upscaled equations. For general micro-scale geometries, the homogenized model that we obtain falls in the class of distributed microstructure models. For circular initial inclusions the distributed microstructure model reduces to system of partial differential equations coupled with an ordinary differential equation. In order to investigate how well the upscaled equations describe the behavior of the micro-scale model, we perform numerical computations for a test problem.

Jan-Marc Vanden Broeck
University of East Anglia

Some New Three Dimensional Free Surface Flows

Over the last 200 years many analytical and numerical results have been obtained for two dimensional free surface flows. A large part of this success is related to the fact that two dimensional free surface flows can be formulated in terms of analytic functions. Relatively few solutions have been calculated for three dimensional free surface flows. In this talk we present new three dimensional free surface flow solutions. These include gravity capillary solitary waves and flows generated by moving disturbances.

Juan Luis Vázquez
Univ Autonoma de Madrid

Porous Medium Flow with Nonlocal Effects

We study a model for flow in porous media including nonlocal diffusion effects. It is based on Darcy'sm law and the pressure is related to the density by

an inverse fractional Laplacian operator. We prove existence of solutions that propagate with finite speed. The model has the very interesting property that mass preserving selfsimilar solutions can be found by solving an elliptic obstacle problem with fractional Laplacian for the pair pressure-density. We use entropy methods to show that the asymptotic behaviour is described after renormalization by these solutions which play the role of the Barenblatt profiles of the standard porous medium model.

Joint work with Luis Caffarelli, Univ. Texas.

Georg S. Weiss
University of Tokyo

A Nonlinear Frequency Formula and the Singular Set of a Free Boundary Problem

The degenerate singular set Σ_z of the Bernoulli free boundary problem is known to be a nullset with respect to the n-dimensional Lebesgue measure. Here we show that Σ_z can be decomposed into a Reifenberg flat part and a lower dimensional nullset. The key is a formula for a nonlinear mean frequency. We discuss generalizations to other equations.

Noemi Wolanski
Universidad de Buenos Aires

Regularity of the Free Boundary for the Limit of an Inhomogeneous Singular Perturbation Problem

In this lecture we present several results on the regularity of the free boundary for the limit $u = \lim u^{\varepsilon_j}$; with $\varepsilon_j \to 0$ and u^{ε_j} solutions to

$$\Delta u^{\varepsilon_j} = \beta_{\varepsilon_j}(u^{\varepsilon_j}) + f_{\varepsilon_j}.$$

where $\beta_{\varepsilon_j} = \frac{1}{\varepsilon_j}\beta\big(\frac{s}{\varepsilon_j}\big)$.

We prove that, under certain assumptions, u is a weak solution to the free boundary problem

$$\begin{cases} \Delta u = f\,\chi_{\{u\not\equiv 0\}} & \text{in } \{u>0\}\cap\{u\le 0\}^\circ \\ u = 0, \quad |\nabla u^+|^2 - |\nabla u^-|^2 = 2M & \text{on } \partial\{u>0\} \end{cases}$$

where $M = \int \beta(s)\, ds$, with β a Lipschitz continuous function, $\beta(s) > 0$ for $s \in (0,1)$, $\beta(s) = 0$ for $s \notin (0,1)$.

The term f_{ε_j} may be a true source term or may come from lower order terms that are already known to be bounded uniformly with respect to ε_j. It may also come from a nonlocal diffusion term.

The proofs make use of the regularity results for two phase "viscosity solutions" of the inhomogeneous stationary problem by Caffarelli, Jerison and Kenig.

Also, we prove a *local* monotonicity formula for the inhomogenous case inspired by a global one by G. S. Weiss for the homogeneous case (this is, $f_{\varepsilon_j} \equiv 0$). This formula is a main tool in some of our results.

Robin Young
University of Massachusetts

Towards Shock-Free Periodic Solutions for the Euler Equations

In this ongoing collaboration with Blake Temple, we attempt to prove the existence of periodic solutions to the Euler equations of gas dynamics. Such solutions have long been thought not to exist due to shock formation, and this is confirmed by the celebrated Glimm-Lax decay theory for 2x2 systems. However, in the full 3x3 system, multiple interaction effects can combine to slow down and prevent shock formation. In this talk I shall describe the physical mechanism supporting periodicity, analyze combinatorics of simple wave interactions, and develop periodic solutions to a "linearized" problem. These linearized solutions have a beautiful structure and exhibit several surprising and fascinating phenomena. I shall also discuss our attempts to prove that these solutions perturb: this leads us to problems of small divisors and KAM theory.

2.2 Contribution Talks

John Andersson
University of Jyväskylä

The Zero Level Set for Certain Weak Solutions with Applications to the Bellman Equations.

We will investigate the zero level set of weak solutions to

$$div(A(u)\nabla u) = 0, \tag{2.2.1}$$

where $A(t) = I$ when $t \geq 0$ and $A(t) = B$ when $t < 0$. Our interest in this problem comes from its relation to the Bellman equations, $\max(\Delta v, div(B\nabla v)) = 0$. In particular, there is an equivalency between the zero level set of solutions to (2.2.1) and the set where v changes phase, from solving $\Delta v = 0$ to solving $div(B\nabla v) = 0$.

We will show that the zero level set has $\sigma-$finite $(n-1)-$dimensional Hausdorff measure and that the zero level set is $C^{1,\alpha}$ almost everywhere with respect to the measure Δu^+.

The proof relies on methods developed by L.A. Caffarelli for two phase free boundary problems and some results by J.M. Marstrand in geometric measure theory.

This is a joint work with H. Mikayeyan.

Teitur Arnarson
KTH, Stockholm

Non-Linear PDE Methods in Terms of Stochastics

We look at methods used in non-linear PDE theory and their applications to finance. The *blow-up technique* for instance can be used to obtain boundary regularity of non-linear free boundary problems.

The PDE problem has a corresponding formulation in terms of stochastic, but we do not find the corresponding method for local analysis of the stochastic problem. Our aim is translate non-linear PDE methods into stochastic language. We show how this is done for the linear optimal stopping problem of pricing American options.

Farid Bozorgnia
KTH, Stockholm

Numerical algorithms for the spatial segregation of competitive systems

A system of m differential equations that appears in population modeling, is considered. At first we study the asymptotic behavior of the positive solutions as the competition rate tends to infinity, then we present various numerical methods.

Cristina Caputo
University of Texas

Motion of Hypersurfaces by the Harmonic Mean Curvature Flow

We will analyze the evolution of weakly convex surfaces in $\mathbf{R}^3$ with flat sides by the harmonic mean curvature flow. We establish short time existence as well as the optimal regularity of the surface and show that the boundaries of the flat sides evolve by the curve shortening flow. It follows from our results that a weakly convex surface with flat sides of class $C^{k,\gamma}$, for some integer $k \geq 1$ and $0 < \gamma \leq 1$, remains in the same class.

Kristoffer J Glover
University of Manchester

Free Boundary Behaviour of Finitely Liquid Markets

A number of models have been proposed with the aim of incorporating finite liquidity and price impact into option pricing theory. For the most part these result in highly nonlinear partial differential equations, on account of the inherent feedback mechanisms involved. We employ novel analytical and numerical techniques, including local similarity analysis, to evaluate the option price behaviour and in particular the behaviour of the early exercise boundary of American style options. Of particular interest is the period close to expiry of the early exercise

options, generally the most intricate time for the free-boundary behaviour. The inclusion of finite liquidity reveals subtle differences from the infinitely liquid Black-Scholes case.

Diogo Aguiar Gomes
Instituto Superior Técnico, Lisbon, Portugal.

Discrete Time, Finite State Space Mean Field Games

We study a mean field model for discrete time, finite number of states, dynamic games. The mean field approach models situations that involve a very large number of agents which move according to certain optimality criteria.

We discuss the set up of the problem, as well as its connection with discrete-time finite number of states optimal control problems. In contrast with optimal control, where usually only the terminal cost V^N is necessary to solve the problem, in mean-field games both the initial distribution of agents π^0 and the terminal cost V^N are necessary to determine the solutions, that is, the distribution of players π^n and value function V^n, for $0 \leq n \leq N$. Because both initial and terminal data needs to be specified, we call this problem the initial-terminal value problem. We establish existence, under quite general conditions, as well as some uniqueness results, both for the stationary and for the initial-terminal value problems. Finally we prove the exponential convergence to a stationary solution of (π^0, V^0), as $N \to \infty$, for the initial-terminal value problem with data π^{-N} and V^N.

This is a joint work with R. R. Sousa and J. Mohr.

Shapour Heidarkhani
University of Mazandaran, Babolsar, Iran

Multiplicity Results to the Existence of Three Solutions for a Class of Neumann Elliptic Systems

In this work, we are interested in multiplicity results for the following Neumann elliptic systems

$$\begin{cases} -\Delta_{p_1} u_1 + a_1(x)|u_1|^{p_1-2}u_1 = \lambda F_{u_1}(x,u_1,...,u_n) + \mu G_{u_1}(x,u_1,...,u_n) \text{ in } \Omega, \\ -\Delta_{p_2} u_2 + a_2(x)|u_2|^{p_2-2}u_2 = \lambda F_{u_2}(x,u_1,...,u_n) + \mu G_{u_1}(x,u_1,...,u_n) \text{ in } \Omega, \\ ... \\ -\Delta_{p_n} u_n + a_n(x)|u_n|^{p_n-2}u_n = \lambda F_{u_n}(x,u_1,...,u_n) + \mu G_{u_1}(x,u_1,...,u_n) \text{ in } \Omega, \\ \frac{\partial u_i}{\partial v} = 0 \text{ for } 1 \le i \le n \text{ on } \partial\Omega \end{cases}$$

where $\Delta_{p_i} u_i = \text{div}(|\nabla u_i|^{p_i-2}\nabla u_i)$ is the p_i-Laplacian operator, $p_i > N$ for $1 \le i \le n$, $\Omega \subset R^N (N \ge 1)$ is non-empty bounded open set with a boundary $\partial\Omega$ of class C^1, $p_i \ge 2$, $a_i \in L^\infty(\Omega)$ with $\text{ess}\inf_\Omega a_i > 0$ for $1 \le i \le n$, $\lambda,\ \mu > 0$, $F : \Omega \times R^n \to R$ is a function such that $F(.,t_1,...,t_n)$ is continuous in Ω for all $(t_1,...,t_n) \in R^n$ and $F(x,.,...,.)$ is $C1$ in R^n for almost every $x \in \Omega$, $G : \Omega \times R^n \to R$ is a function such that $G(.,t_1,...,t_n)$ is measurable in Ω for all $(t_1,...,t_n) \in R^n$ and $G(x,.,...,.)$ is $C1$ in R^n for almost every $x \in \Omega$, and F_{u_i} and G_{u_i} denotes the partial derivative of F and G with respect to u_i, respectively, and v is the outward unit normal to $\partial\Omega$.

Precisely, we deal with the existence of an non-empty open interval $\Lambda \subseteq [0,+\infty[$ and a positive real number q with the following property: for every $\lambda \in \Lambda$ and every $G : \Omega \times R^n \to R$ as above, there exists $\delta > 0$ such that, for each $\mu \in [0,\delta]$, the problem (1) admits at least three solutions in $W^{1,p_1}(\Omega) \times W^{1,p_2}(\Omega) \times ... \times W^{1,p_n}(\Omega)$ whose norms are less than q.

Very recently, B. Ricceri in [2] revisited the three critical points theorem of [1], and our result is fully based on it.

Joint work with G.A. Afrouzi

1 B. Ricceri, *On a three critical points theorem,* Arch. Math. (Basel) 75 (2000) 220-226.

2 B. Ricceri, *On a three critical points theorem revisited,* Nonlinear AnaL. To appear.

Matthieu Hillairet
Université Paul Sabatier

On Collisions Between Rigid Bodies Inside a Viscous Incompressible Fluid

In order to describe interactions between rigid bodies in a viscous incompressible fluid, a common approach consists in writing incompressible Navier Stokes

equations in the fluid domain, with no slip boundary conditions, and applying classical mechanics relations to describe the dynamics of the solids inside the fluid. At the end of the 90s, several studies prove existence of solutions to such problems up to collision between solids (see B. Desjardins and M.J. Esteban, CPDE, (25)2000, pp. 13991413, for example). In the following years, several attempts were made to determine whether these models allow contact or not. First, J.L. Vazquez and E. Zuazua (M3AS, (5), 2006, pp. 637678) prove a no collision result in a simplified 1D problem. Then, V.N. Starovoitov (FBP, Trento, 2002) obtains a criterion for the lack of collision involving sobolev norms of the fluid velocity-field gradient. However, this criterion does not apply a priori to solutions to the Navier-Stokes equations. In my talk, I shall explain how the strategy by V.N. Starovoitov can be improved to obtain a rigorous no collision result and which part of the model may be loosen to allow collisions.

Razvan Gabriel IAGAR
Universidad Autonoma de Madrid

Anomalous Large-Time Behaviour of the P-Laplacian Flow in an Exterior Domain in Low Dimension

We study the large time behavior of weak nonnegative solutions of the p-Laplace equation ($p > 2$) posed in an exterior domain in space dimension $N < p$ with homogeneous Dirichlet boundary conditions, together with the behavior of their free bound- aries. The description is done in terms of matched asymptotics: the outer asymptotic profile is a dipole-like self-similar solution with a singularity at $x = 0$ and anomalous similarity exponents, while the inner asymptotic behavior is given by a separate-variable profile. As intermediate interesting results, we prove a new strong maximum principle for the p-Laplacian equations near degeneracy points. Moreover, we use a fine geo- metric technique to study the evolution of the free boundary, passing through delicate comparison arguments.

Joint work with Prof. Juan Luis Vazquez.

Petra Kordulova
Mathematical Institute in Opava Silesian University in Opava

Preisach Model in Hydrology

We present results about a class of PDEs whose model equation is represented by the Philip-Richards equation with soil-moisture hysteresis term. We assume that the porous media hysteresis is represented by the Preisach hysteresis operator. We introduce a weak formulation of our problem in Sobolev spaces. An existence result is proved by a method based on an ap- proximation of implicit time discretization scheme, a-priori estimates and passage to the limit in the convexity domain of the Preisach operator.

Jimmy Lamboley
ENS Cachan Bretagne, IRMAR, UEB

Regularity of the Boundary of an Optimal Shape With Convex Constraints

We deal with the following shape optimization problem:

$$\lambda_2(\Omega^*) = \min_{\substack{|\Omega|=V_0 \\ \Omega \text{ convex}}} \lambda_2(\Omega),$$

where λ_2 denotes the second eigenvalue of the Laplacian with Dirichlet boundary conditions (in dimension 2).
A. Henrot and E. Oudet studied some properties of optimal shapes Ω^* in [HO]. They prove in particular that the stadium is not optimal. They obtained other geometric properties of Ω^*, under some extra assumptions about the regularity of this one (for example: Ω^* is $\mathcal{C}^{1,1}$).
Precisely, we study here the regularity of the boundary of Ω^*. We will see that Ω^* is at most $\mathcal{C}^{1,1/2}$, and not $\mathcal{C}^{1,1}$ as expected in the work of A. Henrot and E. Oudet. We also explain that, with a few ajustments, the results of [HO] are still available.
Our method can in fact be applied to "partially overdetermined problems", that is to say an equation like:

$$\left\{ \begin{array}{rcccc} -\Delta u & = & f(u) & in & \Omega \\ u & = & 0 & on & \partial\Omega \\ u_\nu & = & -c & on & \Gamma. \end{array} \right.$$

where $\Gamma \subset \partial\Omega$ and $u \in \mathcal{C}^1(\overline{\Omega})$. We will see the link with the previous question, and we will show that our approach carries over some other problems.

HO Henrot A. - Oudet E. - *Minimizing the second eigenvalue of the Laplace operator with Dirichlet boundary conditions,* Arch. for rat. mech. and analysis 2003, vol. 169, **1**, pp. 73-87

FG Fragalà I. - Gazzola F. - Lamboley J. - Pierre M. *C*ounterexamples to symmetry for partially overdetermined elliptic problems , Preprint, 2008

L J. Lamboley, *About Hölder regularity of the optimal convex planar shape for* λ_2, Preprint, 2008

Silvana De Lillo
INFN - Sezione di Perugia

A 2-Phase Free Boundary Problem for a Nonlinear Diffusion-Convection Equation

The Rosen-Fokas-Yorstos equation is a well known model for the two phase flow of immiscible fluids in a porous medium. We consider a 2-phase free boundary problem for such equation and via a contraction mapping technique we show existence and unicity of the solution for short time. An explicit solution is also discussed

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Erik Lindgren
KTH, Stockholm

On the Two-Phase Obstacle Problem with Holder Continuous Coefficients

We study the regularity of two-phase obstacle problem, i.e.

$$\Delta u = \lambda_1 \chi_{\{u>0\}} - \lambda_2 \chi_{\{u<0\}} \text{ in } B_1$$

with coefficients λ_i that merely Holder continuous. When the coefficients are assumed to be Lipschitz one can apply the monotonicity formula to prove $C^{1,1}$-regularity of the solution and also the C^1-regularity of the free boundary near so called branching points. However, there is no available monotonicity formula in this case. Instead we use a specific scaling argument to obtain $C^{1,1}$-estimates and

then the stability in coefficients to prove the C^1-regularity of the free boundary near so called branching points.

This is an ongoing project with Anders Edquist and Henrik Shahgholian, both at KTH.

Sandra Martinez
Universidad de Buenos Aires

A Singular Perturbation Problem for a Quasilinear Operator Satisfying the Natural Growth Condition of Lieberman

We study the following problem. For any $\varepsilon > 0$, take u^ε a weak solution of

$$\mathcal{L}u^\varepsilon := \operatorname{div}\Big(\frac{g(|\nabla u^\varepsilon|)}{|\nabla u^\varepsilon|}\nabla u^\varepsilon\Big) = \beta_\varepsilon(u^\varepsilon), \quad u^\varepsilon \geq 0.$$

Here $\beta_\varepsilon(s) = \frac{1}{\varepsilon}\beta\left(\frac{s}{\varepsilon}\right)$, with $\beta \in \mathrm{Lip}(\mathbf{R})$, $\beta > 0$ in $(0,1)$ and $\beta = 0$ otherwise.

We assume that g satisfies the conditions introduced by G. Lieberman in [Li1]. The conditions on the function g allow for a different behavior at 0 and at ∞. Moreover the set of functions that satisfy these conditions include inhomogeneous functions.

We are interested in the limiting problem, when $\varepsilon \to 0$. As in previous work with $\mathcal{L} = \Delta$ or $\mathcal{L} = \Delta_p$ we prove, under appropriate assumptions, that any limiting function is a weak solution to a free boundary problem (in the sense defined in [MW1]). Moreover, for nondegenerate limits we prove that the reduced free boundary is a $C^{1,\alpha}$ surface. This result is new even for Δ_p. Finally, we give two examples in which we can apply the regularity results. In both examples the nondegeneracy property is satisfied by the limiting function.

Li1 G. M. Lieberman, *The natural generalization of the natural conditions of Ladyzhenskaya and Ural'tseva for elliptic equations*, Comm. Partial Differential Equations **16** (1991), no. 2-3, 311–361.

MW1 S. Martínez and N. Wolanski, *A minimum problem with free boundary in Orlicz spaces*, Adv. Math. (2008).

Fernando Miranda
Univ Minho, Braga

A New Quasi-Variational Inequality for a Maxwell System

We consider a stationary non-homogeneous electromagnetic problem motivated by the Bean's critical-state model for superconductors.

We introduce the natural functional framework composed by divergent-free L^p functions having curl in L^p and null normal-trace and we prove a Poincaré-Friedrichs type inequality.

Imposing that the current density does not exceed some critical value, that may depend on the magnetic field, the model becomes a quasi-variational inequality for which we are able to prove existence of a solution.

(Joint work with José Francisco Rodrigues and Lisa Santos.)

Hideki Murakawa
University of Toyama

A Solution of Parabolic Free Boundary Problems by Semilinear Reaction-Diffusion Systems

A nonlinear degenerate parabolic problem of Stefan and porous medium type is considered. The degeneracy of the diffusion characterizes the presence of a free boundary between different phases and the solution exhibits a global lack of regularity across the free boundary. Therefore, analysis and numerical approximation of the problem are much harder than those of mildly nonlinear problems and semilinear problems. To avoid the nonlinearities of the diffusion with degeneracy of the diffusion, we propose a reaction-diffusion systems with solutions that approximate those of the nonlinear problems. The reaction-diffusion system includes only a simple reaction and linear diffusion. Resolving semilinear problems is typically easier than dealing with nonlinear diffusion problems such as the degenerate parabolic equations. Therefore, our ideas are expected to reveal new and more effective approaches to the study of nonlinear diffusion problems. We also consider application of the reaction-diffusion system to numerical methods for the nonlinear degenerate parabolic problem. A discrete-time scheme for the nonlinear problem is proposed by means of the reaction-diffusion system. We can obtain stability results and the optimal error estimates for the scheme. Numerical experiments are carried out using a numerical algorithm based on the

discrete-time scheme. The numerical results show the effectiveness and efficiency of the algorithm.

Yuliya V. Namlyeyeva
Institute of Applied Mathematics and Mechanics of NAS of Ukraine

Asymptotic Behavior of Solutions to the Dirichlet Eigenvalue Problem in Perforated Domains of General Structure

We study the asymptotic behavior of the solutions to the Dirichlet eigenvalue problem for the linear higher order elliptic operator in the sequence of perforated domains. No more complication arises from the complex structure of the domain in the proof of the solvability of this problem but it is practically impossible to find its solution neither by analytical no by numerical methods. In this case the homogenization methods are usually used. Under several conditions on the perforated domain, one can describe the leading term of the asymptotic expansion of the solution of the initial problem in terms of solution of a new problem considered in a simple domain.

The homogenization result for the linear higher order elliptic problem in domains of general structure was obtained in [1]. The asymptotic behavior of the Dirichlet problem for nonlinear elliptic second order equations under sufficiently weak assumptions on perforated domain was obtained in [2]. The Dirichlet problem for nonlinear higher order equations with a special ellipticity condition and without any geometrical assumptions on the structure of perforation was studied in [3].

In this talk we consider a sequence of perforated domains of a general structure. The assumptions on the perforations are formulated in terms of Borel's measures. Following the lines of [1, 2], we construct the limit problem for the leading term of the asymptotic expansion of the solution to the Dirichlet eigenvalue problem for linear higher order elliptic equations in perforated domains.

Joint work with Šárka Nečasová.

1 Khruslov, E.Ya., The first boundary value problem in domains with a complicated boundary for higher order equations, *Mat. Sbornik* **103(145)**, N 4(8), (1977), 614–629.

2 Dal Maso, G., Skrypnik, I.V., Asymptotic behaviour of nonlinear Dirichlet problem in perforated domains, *Ann. Mat. Pura Appl.* IV, Ser.174 (1998), 13–72.

3 Dal Maso, G., Skrypnik, I.V., Asymptotic behaviour of nonlinear elliptic higher order equations in perforated domains, *J. Anal. Math.* **79**, (1999), 63–112.

Kaj Nyström
Umeå University

Regularity of Free Boundaries in Two-phase Problems for the p-Laplace Operator

Some time ago L. Caffarelli developed, in a sequence of three papers, a celebrated theory for general two-phase free boundary problems for the Laplace operator. In the first paper Lipschitz free boundaries were shown to be $C^{1,\gamma}$-smooth for some $\gamma \in (0,1)$ and in the second paper it was shown that free boundaries which are well approximated by Lipschitz graphs are in fact Lipschitz. Finally, in the third paper the existence part of the theory was developed. Recently John Lewis and I have been able to generalize the results in the first two papers to the p-Laplace operator when $p \neq 2$, $1 < p < \infty$, and the purpose of this talk is to briefly describe these results. Our generalizations beyond the harmonic case, which corresponds to $p = 2$, are non-trivial due to the non-linear and degenerate character of the p-Laplace operator. In particular, our results and arguments rely heavily on a toolbox of techniques which John Lewis and I have developed in our studies of the boundary behaviour of p-harmonic functions in Lipschitz domain and in domains which are well approximated by Lipschitz domains in the Hausdorff distance sense. In particular, our results concerning general two-phase free boundary problems for the p-Laplace operator represent a 'tour de force' of the techniques we have developed.

David Siegel
University of Waterloo

Behaviour of a Capillary Surface Near a Cusp

Consider two tangent vertical cylinders, of possibly different materials, immersed in a fluid bath. We investigate the resultant equilibrium capillary surface near the cusp. When the contact angles on the two cylinders are supplementary, then the solution is continuous up to the corner. When the contact angles are not supplementary then the solution tends to plus or minus infinity and we determine the order of growth. Previously, Markus Scholz (2003) studied the case of power law cusps with non-supplementary contact angles.

Domingo A. Tarzia
Depto. Matemática-CONICET, FCE, Univ. Austral

The Coupled Heat and Mass Transfer During the Freezing and Sublimation Processes of High-Water Content Food Materials

A coupled problem of heat and mass transfer during the solidification of high-water content materials (e.g. foods, moist soils, acqueous solutions and vegetable or animal tissues) is analyzed. When these kinds of materials are refrigerated two simultaneous physical phenomena take place: liquid water solidifies (freeze), and surface ice sublimates. Ice sublimation takes place at the surface of high-water content systems when they are frozen uncovered or without an impervious and tight packaging material. The rate of both phenomena (solidification and sublimation) is determined both by material characteristics and cooling conditions. The sublimation process determines fundamental features of the final quality of foods and influences the structure and usefulness of frozen tissues. Modeling these two simultaneous processes is very difficult owed to the coupling of the heat and mass transfer balances as well as to the existence of two phase-change fronts that advance with very different velocities.
The process when only a solidification front is present has been extensively studied in literature. On the contrary, mathematical results in the case in which both freezing and ice sublimation appear are scarce and no analytical solution has been found. Moreover no experimental data about temperature and concentration profiles, and freezing and dehydration fronts are available. On the other hand, theoretical models were proposed to describe the heat and mass transfer in these processes but, in most of the published works, only semi-empiric or numerical methods are used to investigate the properties of solutions.
As well as the description of the freezing process is concerned, the material can be divided into three zones: unfrozen, frozen and dehydrated. Freezing begins at the refrigerated surface, simultaneously, ice sublimation begins at the frozen

surface and a dehydration front penetrates the material. Normally dehydration velocity is much lower than that of the freezing front.
We consider a semi-infinite material with characteristics similar to a very diluted gel (whose properties can be supposed equal to those of pure water). The system has an initial uniform temperature equal to T_{if} and an uncovered flat surface which, at time $t = 0$, is exposed to the surrounding medium (with constant temperature T_s lower than T_{if} , and heat and mass transfer coefficients h and K_m). We assume that $T_s < T_0(t) < T_{if}$ where $T_0(t)$ is the unknown sublimation temperature. In order to calculate the evolution of temperature and water content in time, we will consider a two-phase free boundary problem for the temperatures $T_d = T_d(x,t)$ (dehydrated region) and $T_f = T_f(x,t)$ (frozen region), the vapour concentration $C = C(x,t)$ (dehydrated region), the free boundaries $s_d = s_d(t)$ (sublimation front) and $s_f = s_f(t)$ (frozen front) and the temperature $T_0(t)$ at the sublimation front $x = s_d(t)$.
We developed a quasi-analytical model under few simplifying assumptions, thus obtaining a set of coupled ordinary differential equations which enables to predict, easily, the influence of material characteristics and freezing conditions on the evolution of both frozen and sublimation fronts. In the same way we are able to find the temperature in the frozen and dehydrated zones, the temperature at the sublimation front and the vapour concentration in the dehydrated layer. Such results were validated against the analytical solution describing the freezing process in a semi-infinite material when the sublimation does not occur. Our quasi-analytical model has been extensively used to perform a parametric study of the problem and, in the future, it will be used to optimize the freezing conditions of certain types of materials.
This is a joint paper with Rodolfo H. Mascheroni, Mariela C. Olguín and Viviana O. Salvadori.

Jacques du Toit
University of Manchester

Selling a Stock at the Maximum

Imagine an investor who owns a stock and wishes to sell it before time $T > 0$ so as to maximize his profit. The investor has to decide when to sell the stock. Naturally, he would like to sell when the stock price is at its maximal value over the interval $[0, T]$, but such a strategy is impractical since this information is only known at time T . What the investor would like to do at any time $t[0, T]$

is to use all the accumulated information to infer how close the stock price is to the ultimate maximum, and based on this decide whether he should sell or not. The investor is thus faced with solving an *optimal prediction problem* a type of optimal stopping problem where the gain function depends on the future and is only $\mathcal{F}_T$ measurable.

We will present a solution to this problem when the stock price follows geometric Brownian motion. The optimal prediction problem naturally leads to a *free boundary problem*, and we will discuss how local timespace calculus techniques from probability theory can be used to characterize the free boundary and the value function.

This is joint work with Prof. Goran Peskir.

Eugen Varvaruca
University of Bath

On the Existence of Extreme Waves and the Stokes Conjecture With Vorticity

We present some recent results on singular solutions of the problem of traveling gravity water waves on flows with vorticity. We show that, for a certain class of vorticity functions, a sequence of regular waves converges to an extreme wave with stagnation points on the free surface. We also show that, for any vorticity function, the profile of an extreme wave must have either a corner of 120° or a horizontal tangent at any stagnation point about which it is supposed symmetric. Moreover, the profile necessarily has a corner of 120° if the vorticity is nonnegative near the free surface.

Yasunori Aoki
University of Waterloo

An Approximation Technique for the Laplace-Young Capillary Equations in a Circular Cusp Region

Solder plating of a semiconductor pin is one of the industrial applications of capillary free boundary problems. The capillary filling in a circular cusp region has been explored employing the Laplace Young equations. In this problem, the

tangent cylindrical coordinate system has shown a significant advantage over the previously used curvilinear coordinate system. In order to approximate the solution to the Laplace Young equations, modified Laplace Young Equations are constructed. It can be shown that the use of the tangent cylindrical coordinate system make it possible to solve the modified Laplace Young equations exactly. Also the exact solution to the modified Laplace Young equations is shown to be a seventh order accurate asymptotic solution to the original Laplace Young equations. This talk will address how a slight modification to the PDE problem and a proper choice of the coordinate system dramatically simplified a problem and unveiled an accurate approximation.

Erwin Vondenhoff
Eindhoven University of Technology

Long-Time Asymptotics for Some Hele-Shaw Models With Injection or Suction in a Single Point

We discuss longtime behavior of Hele-Shaw flow with injection and suction in a single point, for domains that are small perturbations of balls. An evolution equation for the motion of these domains is derived and linearized. We use spectral properties of the linearization of this equation to show that in the case of injection, perturbations of balls decay algebraically. If for the three-dimensional case surface tension is included, all liquid can be removed by suction if the suction point and the geometric centre coincide and the ratio of suction speed and surface tension is small enough. We also show the existence of noncircular, self-similar solutions up to complete extinction. These solutions are found as bifurcation solutions to a nonlocal elliptic equation of order three.
