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Singularities of axisymmetric free surface flows with gravity

In this talk we present some recent results, obtained in collaboration with Georg Weiss, on singularities of steady axisymmetric solutions of the Euler equations for a fluid (incompressible and with zero vorticity) with a free surface, acted on only by gravity. We use geometric methods to analyze the asymptotics of the velocity field and of the free surface at stagnation points as well as at points on the axis of symmetry.
At points on the axis of symmetry which are not stagnation points, *constant velocity motion* is the only blow-up profile consistent with the invariant scaling of the equation. This suggests the presence of downward pointing cusps at those points.
At *stagnation points* on the axis of symmetry, the unique blow-up profile consistent with the invariant scaling of the equation is *Garabedian's pointed bubble solution* with water above air. Thus at stagnation points on the axis of symmetry with no water above the stagnation point, the invariant scaling of the equation cannot be the right scaling. A fine analysis of the blow-up velocity yields that in the case that the surface is described by an injective curve, the velocity scales almost like $\sqrt{X^2+Y^2+Z^2}$ and is asymptotically given by the velocity field

$$V(\sqrt{X^2+Y^2}, Z) = c(-\sqrt{X^2+Y^2}, 2Z)$$

with a nonzero constant c.
The last result relies on a frequency formula in combination with a concentration compactness result for the axially symmetric Euler equations by Delort. While the concentration compactness result alone does *not* lead to strong convergence in general, we prove the convergence to be strong in our application.