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**Arbitrary Lagrangian-Eulerian finite-element method for  
computation of two-phase flows with soluble surfactants**

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A finite-element scheme based on a coupled arbitrary Lagrangian-Eulerian and Lagrangian approach is developed for the computation of interface flows with soluble surfactants. The numerical scheme is designed to solve the time-dependent Navier-Stokes equations and an evolution-equation for the surfactant in the bulk phase, and simultaneously, an evolution-equation for the surfactant on the interface. For this second order isoparametric finite-elements on moving meshes and second order isoparametric surface finite-elements have been used. The interface resolved moving meshes allow us to incorporate surface forces, Marangoni forces and jumps in the material parameters accurately. The lower dimensional finite-element meshes for solving the surface evolution-equation are part of the interface resolved moving meshes. The numerical scheme is validated for problems with known analytical solutions. A number of computations to study the influence of the surfactants in 3D-axisymmetric rising bubbles have been performed. The proposed scheme shows excellent conservation of fluid mass and the total mass of the surfactant.

**References**

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