Elisabetta Rocca

Weak formulations of PDEs in thermomechanics

Recently, in cooperation with E. Feireisl, H. Petzeltova, R. Rossi, and G. Schimperna, we faced the problem of developing new mathematical techniques and solution notions for some nonlinear evolutionary PDEs, which arise from modeling approaches and able to capture as many aspects of the thermomechanical process as possible.
We basically reinterpret both the concept of weak solution satisfying a suitable energy conservation and entropy inequality - recently introduced by E. Feireisl for a problem of heat conduction in fluids - and the notion of energetic solution introduced by A. Mielke and co-authors in the framework of rate-independent processes.
These ideas open new horizons in the analysis of highly nonlinear and possibly degenerating PDEs, arising from different applications such as the ones related to mixtures of fluids, viscoelastic materials, SMA, and liquid crystal flows.
We will show in the lecture how these solution notions turn out to be very convenient. It is mainly for three reasons: they comply with thermodynamical principles and hence they give for free thermodynamically consistent models; they require the minimal regularity properties of solutions, which is just the one provided by the energy-entropy estimates; they are consistent with the classical solution notion in case one is able to prove further regularity properties of solutions.