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An Adaptive Two-Scale Model for Phase Transitions with Evolution of Microstructures

We develop a fast and accurate adaptive algorithm that numerically solves a two-scale model describing a phase transition of a binary mixture with the evolution of equiaxed dendritic microstructures.
The two-scale model consists of a macroscopic heat equation and a family of microscopic cell problems that model the phase transition of a binary mixture. Both scales are coupled; the macroscopic temperature influences the evolution of the microstructure and the microscopic fields enter to the macroscopic heat equation via averaged coefficients. Adaptivity exploits the fact that the evolving microstructure depends in a continuous way on the macroscopic temperature field; we use the same microscopic data for macroscopic nodes with similar temperature evolutions. We define a metric that compares temperature evolutions and invent adaptive strategies to select active macroscopic nodes. Microscopic cell problems are solved for active nodes only; microscopic data in inactive nodes is approximated from microscopic data of active nodes with similar temperature evolution. The set of active nodes is updated online: it is coarsened until all active nodes have different temperature evolu tions and it is refined until every inactive node has at least one active node with a similar temperature evolution.
We present numerical examples which show that the adaptive method is almost as accurate as the direct solution of the two-scale model while being computationally much more efficient.