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A generalized Lagrange multiplier problem with nonconstant gradient constraint

In this talk we prove the existence of $(u, \lambda) \in W^{1,\infty}(\Omega) \times L^\infty(\Omega)'$, solution of the Lagrange multiplier problem

$$\begin{aligned} -\nabla \cdot \big(\lambda \nabla u\big) = f \ \ &\text{in } \mathcal{D}'(\Omega), \\ u = 0 \ \ &\text{on } \partial\Omega, \\ |\nabla u| \leq \varphi \ &\text{a.e. in } \Omega, \\ \lambda \geq 1, & \\ (\lambda - 1)(|\nabla u| - \varphi) = 0 \ \ &\text{in } \mathcal{D}'(\Omega), \end{aligned}$$

for $f \in L^2(\Omega)$ and $\varphi \in W^{2,\infty}(\Omega)$ positive.
It is well known that if (u, λ) solves this problem then u solves the variational inequality

$$\int_\Omega \nabla u \cdot \nabla(v - u) \geq \int_\Omega f(v - u), \quad \forall v \in \mathbb{K}_\varphi^\nabla,$$

where $\mathbb{K}_\varphi^\nabla = \{v \in H_0^1(\Omega) : |\nabla v| \leq \varphi \text{ a.e. in } \Omega\}$.

(Joint work with Assis Azevedo and Lisa Santos)