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Existence, Uniqueness, and Regularity for Degenerate Obstacle Problems in Mathematical Finance

The Heston stochastic volatility process, which is widely used as an asset price model in mathematical finance, is a paradigm for a degenerate diffusion process where the degeneracy in the diffusion coefficient is proportional to the square root of the distance to the boundary of the half-plane. The generator of this process with killing, called the elliptic Heston operator, is a second-order degenerate elliptic partial differential operator whose coefficients have linear growth in the spatial variables and where the degeneracy in the operator symbol is proportional to the distance to the boundary of the half-plane. With the aid of weighted Sobolev spaces and Holder spaces defined by suitable metrics, we prove existence, uniqueness, and regularity of solutions to obstacle problems for the Heston operator on unbounded subdomains of the half-plane. In mathematical finance, solutions to obstacle problems for the elliptic Heston operator correspond to value functions fo r perpetual American-style options on the underlying asset.
This is joint work with Panagiota Daskalopoulos (Columbia University) and Camelia Pop (Rutgers University).