Numerical investigation of eigenvalues of ∞-Laplacian operator

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Let Ω be a bounded domain in $\mathbb{R}^n$. Eigenvalues of infinity Laplacian are defined as limit of eigenvalues of $p-$Laplace operator. Consider the following minimization problem which is called Rayleigh quotient

$$\min_{u\in W_0^{1,p}(\Omega)} \frac{\int_\Omega |\nabla u|^p dx}{\int_\Omega |u|^p dx} \quad 1 < p < \infty.$$

The corresponding Euler-Lagrange equation is

$$\operatorname{div}(|\nabla u|^{p-2}\nabla u) + \lambda |u|^{p-2} u = 0. \tag{1}$$

Let u_p be the unique positive solution of equation (1) which satisfies

$$\int_\Omega |u_p|^p dx = 1.$$

If u_∞ is a limiting point of $\{u_p\}$, i.e., there exists a subsequence $p_j \to \infty$ such that

$$u_{p_j} \to u_\infty \text{ uniformly in } \overline{\Omega}.$$

Then in [1, 2] it is shown that u_∞ is a viscosity solution of

$$\min\{|\nabla u| - \Lambda u, -\Delta_\infty u\} = 0. \tag{2}$$

Let ∇u and $D^2 u$ denote the gradient and Hessian of u, respectively, and $F_\Lambda(x, u, \nabla u, D^2 u)$ be a continuous real valued function defined on $\Omega \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{S}^n$, with $\mathbb{S}^n$ being the space of symmetric $n \times n$ matrices. Let define the function F_Λ by

$$F_\Lambda(x, u, \nabla u, D^2 u) = \begin{cases} \min\{|\nabla u| - \Lambda u, -\Delta_\infty u\} & u > 0, \\ -\Delta_\infty u & u = 0, \\ \max\{|\nabla u| - \Lambda u, -\Delta_\infty u\} & u < 0. \end{cases} \tag{3}$$

A non-zero function $u \in C(\overline{\Omega}), u = 0$ on $\partial\Omega$, is called an ∞-eigenfunction, if there exist $\Lambda \in \mathbb{R}$ such that the following holds in viscosity sense

$$F_\Lambda(x, u, \nabla u, D^2 u) = 0, \quad \text{in } \Omega.$$

In this talk, a convergent finite difference scheme to approximate the first and the second eigenfunctions defined by equations (2) and (3) and corresponding eigenvalues is given.

REFERENCES

[1] P. Juutinen and P. Lindqvist, On the higher eigenvalues for the ∞- eigenvalue problem, Calc. Var. Partial Differential Equations 23 (2005), 169-192.

[2] P. Juutinen, P. Lindqvist, and J. J. Manfredi, The ∞-eigenvalue problem, Arch. Ration. Mech. Anal. 148 (1999), 89-105,