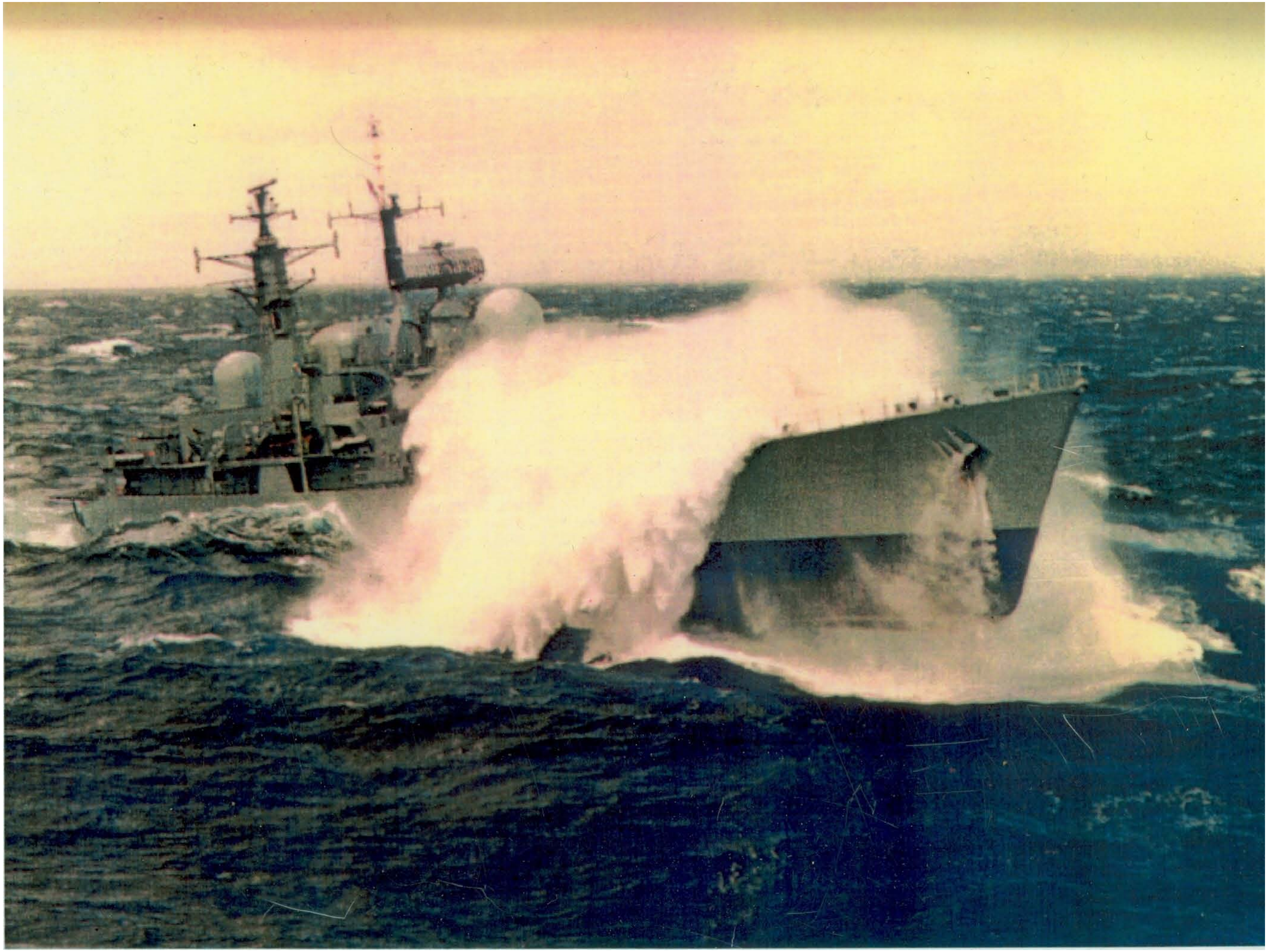


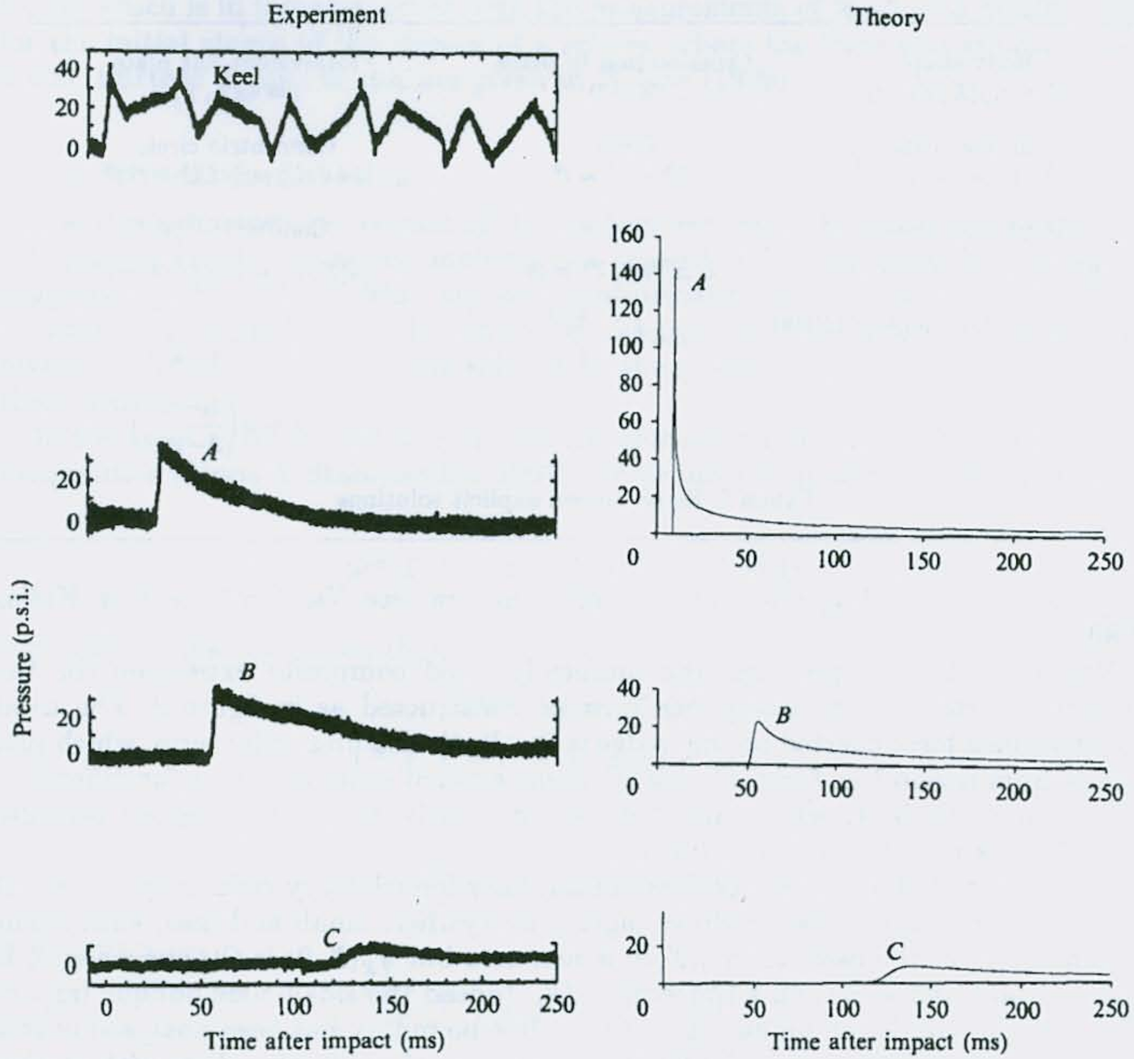
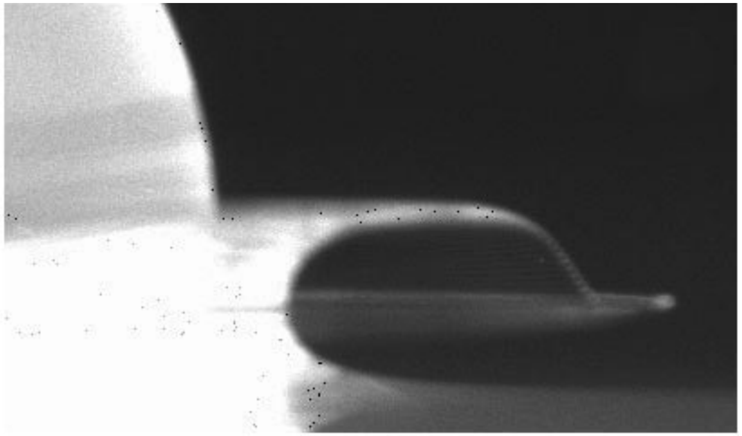
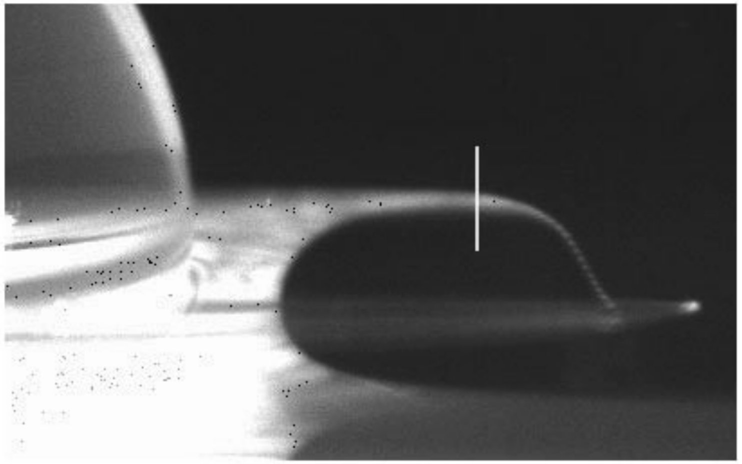


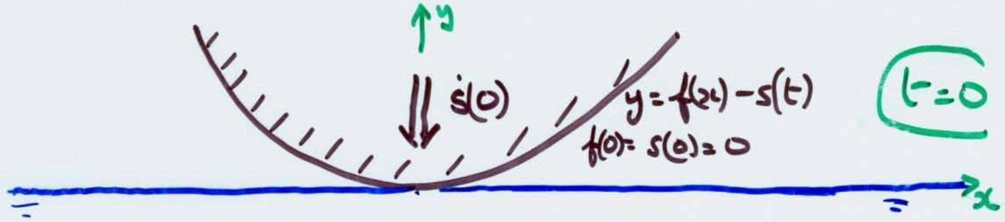
FIGURE 3. Comparison of experimental and theoretical pressure histories. Experimental data from Nethercote *et al.* (1986) with $L = 12$ ft, $V = 20$ ft/s and $\epsilon = 0.707$ so that the dimensionless hull shape is $y = (0.707x)^2$. At the keel the theory predicts an infinite pressure and the oscillations observed experimentally are probably due to air entrapment.





SIMPLEST 2-D DIMENSIONLESS IMPACT MODEL:

GRAVITY = 0 ✓   VISCOSITY = 0 ✓   SURFACE TENSION = 0 ?



$$\frac{\partial \phi}{\partial n} = -\dot{s}\, n_y$$

$$(n_x, n_y) = (-f', 1)/\sqrt{1+f'^2}$$

on $y = f(x) - s(t)$

$$\Delta \phi = 0$$

$$\frac{\partial \phi}{\partial n} = V_n$$

AND $\dfrac{\partial \phi}{\partial t} + \dfrac{1}{2} |\nabla \phi|^2 = 0$

on $y = h$

$$|\nabla \phi| \to 0 \quad \text{at } \infty$$

$$h(x, 0) = \phi(x, y, 0) = 0$$

6

SIMPLEST GEOMETRY IS <u>WEDGE</u>:

$$f(x) = \alpha |x|$$

THEN, IF $s = t$, CLASSICAL $\boxed{\text{WEDGE ENTRY PROBLEM}}$

HAS SIMILARITY SOLUTION

$$\varphi = t\,\hat{\varphi}(x/t, y/t), \quad h = t\,\hat{h}(x/t).$$

MOREOVER IT CAN BE REDUCED TO A <u>1-D</u> INTEGRAL EQUATION USING

WAGNER TRANSFORMATION FOR

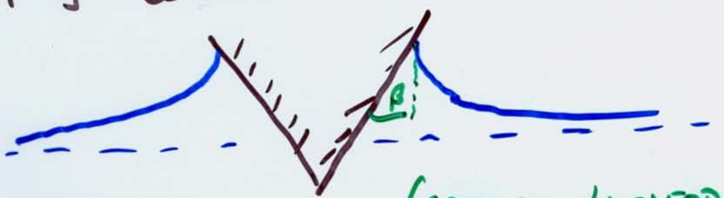
$$w(z) = \hat{\varphi} + i\hat{\psi} :$$

FLOW DOMAIN IN $\int_0^z \sqrt{w''(\zeta)}\, d\zeta$ PLANE

IS AND CAN BE MAPPED ONTO w.

MACKIE
GARABEDIAN
DOBROVOLSKAYA
⋮

BUT $\exists$ CONTINUING CONTROVERSY ABOUT β:



(FRAENKEL/McLEOD 1990-2012)

TO STUDY SPLASHING, WE MAKE A DIFFERENT GEOMETRIC SIMPLIFICATION:

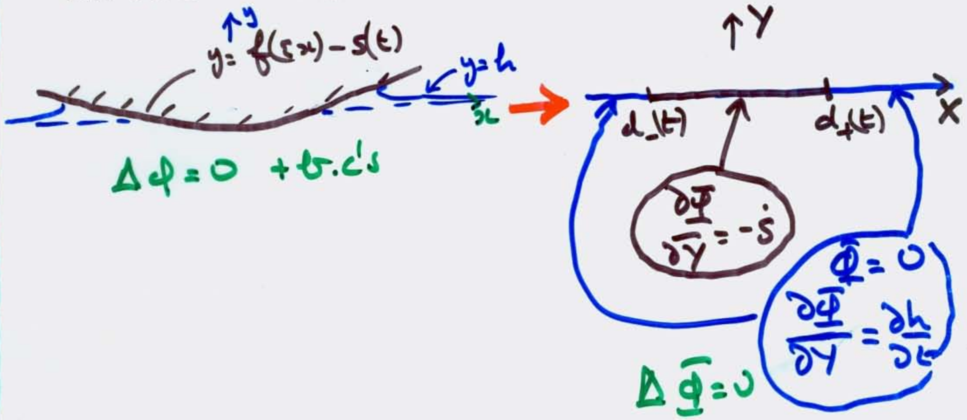
WRITE $f = f(\varepsilon x)$ WHERE $0 < \varepsilon \ll 1$,

ε IS THE DEADRISE ANGLE.

THIS IS THE REGIME IN WHICH DAMAGE CAN BE DONE TO SHIPS, STOMACHS ETC.

(ALSO MODELS SKIMMING)

WAGNER WAS FIRST TO SUGGEST THAT



WHEN WE WRITE

$\phi \to \varepsilon^{-1}\Phi$, $(x,y) \to \varepsilon^{-1}(X,Y)$ ($\Rightarrow$ pressure $= O(\varepsilon^{-1})$)

i.e. REDUCTION TO COD-2 FBP. ON $Y=0$.

"EXPANDING PLATE MODEL" BUT HOW TO FIND $d_\pm$?

IN 1982, KOROBKIN TRANSFORMED TO

DISPLACEMENT POTENTIAL

$$\bar{\Phi}^{*}(x,y,t) = \int_0^t \phi(x,y,\tau)\,d\tau \quad \text{(cf. BAIOCCHI)}$$

TO GIVE

d_- $\qquad$ d_+

$x = d(t) \Updownarrow t = \omega(x)$

$\dfrac{\partial \bar{\Phi}^{*}}{\partial y} = f - s$ FOR $d_- < x < d_+$ $(t > \omega(x))$

$\left.\begin{array}{l} \bar{\Phi}^{*} = 0 \\ \dfrac{\partial \bar{\Phi}^{*}}{\partial y} = h \end{array}\right\}$ FOR $x > d_+$ OR $t < \omega(x)$

$$\Delta \bar{\Phi}^{*} = 0$$

$$\Rightarrow \quad \bar{\Phi}^{*}\left(\frac{\partial \bar{\Phi}^{*}}{\partial y} - f + s\right) = 0 \quad \text{ON } y = 0$$

WITH $\dfrac{\partial \bar{\Phi}^{*}}{\partial y} \le f - s$, $\quad \bar{\Phi}^{*} \le 0$ ON $y=0$.

(FREE SURFACE DOES NOT PENETRATE IMPACTOR) $\qquad$ (PRESSURE ≥ 0)

THIS IS A GREAT HELP NUMERICALLY,

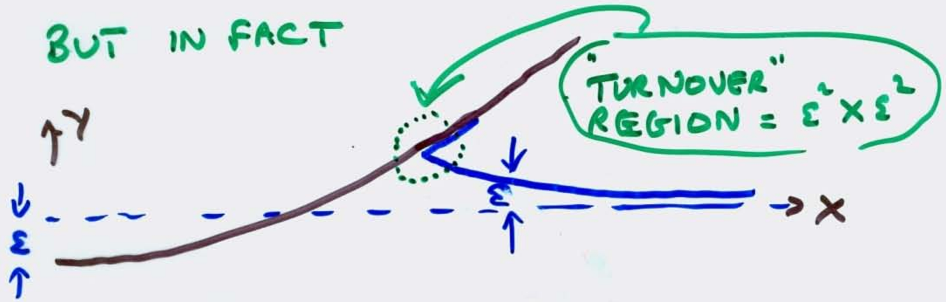
BUT IT RELIES ON

$$\frac{\partial \bar{\Phi}^{*}}{\partial y} \to -s + f \quad \text{AS } t \to \omega.$$

WHY?

WE NEED MAGNIFYING GLASS (M.A.E's) TO EXAMINE THE BEHAVIOUR NEAR THE COD-2 F.B. IT IS TEMPTING TO THINK THAT

BUT IN FACT



WHICH IS THE ASYMPTOTIC REASON FOR THE WAGNER EXPANDING PLATE CONDITION $h(d_+,t) = f(d_+,t) - s(t)$.

MOREOVER THE SCALINGS

$$X = d_+ + \varepsilon^2 \tilde{X}, \quad Y = \varepsilon\, h(d_+,t) + \varepsilon^2 \tilde{Y},$$

$$\Phi = \dot{d}_+ \tilde{X} + \tilde{\Phi}(\tilde{X},\tilde{Y})$$ (WAVE TRAVELLING WITH $\dot{d}_+$)

$\Rightarrow$ PRESSURE $\sim \varepsilon^{-2}$ IN TURNOVER REGION

ALSO, SINCE SOLUTION OF COD-2 F.B.P. AS A MIXED BOUNDARY VALUE PROBLEM IS

(WITH $s = t$, $d_+ = -d_-$ FOR SIMPLICITY)

$$\Phi = \mathrm{Re}\left\{-\left(Y + \left(d^2 - (X+iY)^2\right)^{1/2}\right)\right\},$$

(NOTE $|\nabla \Phi| \sim |d - (X+iY)|^{-1/2}$ NEAR TURNOVER)

WE FIND THAT $d(t)$ SATISFIES

$$f(d(t)) - t = \int_0^t \left(-1 + \frac{d(\tau)}{\sqrt{d^2(t) - d^2(\tau)}}\right) d\tau$$

$$\Rightarrow \quad d^{-1}(X) = \frac{2}{\pi}\int_0^X \frac{f(\xi)\, d\xi}{\sqrt{X^2 - \xi^2}}$$

HENCE FOR WEDGE $d \propto t$

PARABOLA $d \propto t^{1/2}$

⋮

TO LOWEST ORDER, THE TURNOVER REGION IS MODELLED BY THE STEADY

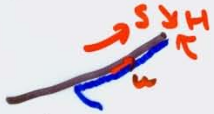
HELMHOLTZ FLOW :

$\Delta \tilde{\Phi} = 0$

$$\frac{\partial \tilde{\Phi}}{\partial n} = 0$$
$$\frac{1}{2}|\nabla \tilde{\Phi}|^2 = \frac{1}{2}\dot{d}_t^{\,2}$$

WITH $|\nabla \tilde{\Phi}| = O(r^{-1/2})$ AT ∞.

HENCE $\exists$ A SPLASH JET (OF THICKNESS $O(\varepsilon^2)$ AND LENGTH $O(\varepsilon^{-1})$ IN X, Y) MODELLED BY "ZERO GRAVITY SHALLOW WATER EQUATIONS



$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial s}(uH) = 0$$
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} = 0$$

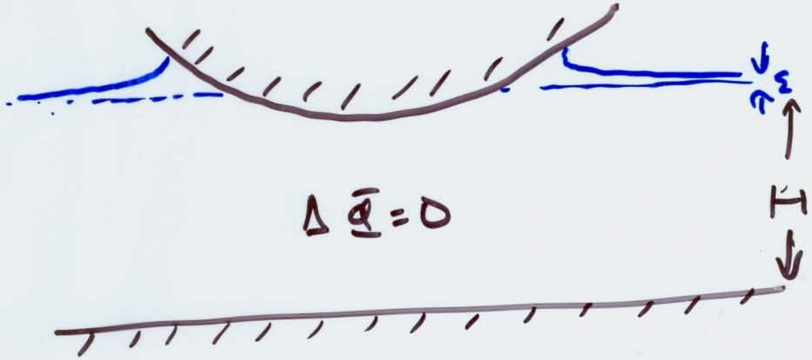
(UNTIL GRAVITY TAKES OVER)

BUT FOR THIS THEORY TO HAVE MUCH PRACTICAL APPLICABILITY WE MUST CONSIDER

- FINITE DEPTH EFFECTS
- OBLIQUE IMPACT
- NON-SMOOTH IMPACTORS
- AIR "CUSHIONING".

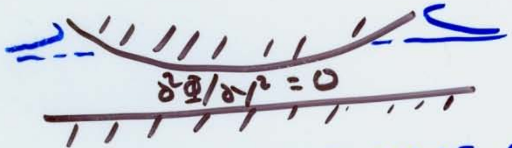
FINITE DEPTH EFFECTS

(i) PENETRATION $\ll$ DEPTH:



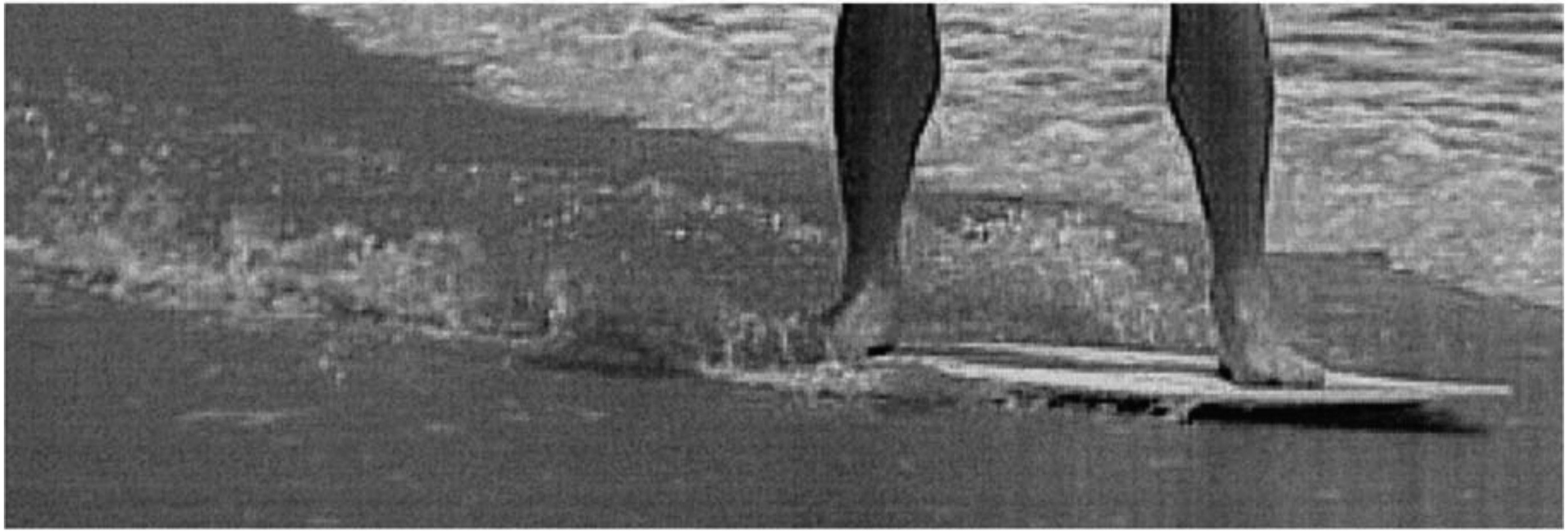
WAGNER THEORY STILL APPLIES
BUT WITH MORE COMPLICATED COD-2 PROBLEM

(ii) PENETRATION $\sim$ DEPTH (KOROBKIN)



WE NO LONGER HAVE TO SOLVE A POTENTIAL PROBLEM ON THE "OUTER" SCALE, BUT TURNOVER PROBLEM IS THE HELMHOLTZ PROBLEM FOR A "SKIMMER"

STRONG JET



REMARK: FOR DROPLET IMPACT ON A SHALLOW LAYER, THE COD-2 F.B.P.S ARE

(i)

(ii)

AND HELMHOLTZ FLOW CAN BE

FIRST

THEN

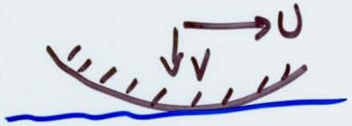
SO JET COMPOSITION CHANGES AS t INCREASES.

OBLIQUE ENTRY

THE SCALING ARGUMENT

$\phi \sim \varepsilon^{-1}, \quad x, y \sim \varepsilon^{-1}$

IS NOT AFFECTED BY U UNTIL



$U \sim \varepsilon^{-1} V = \varepsilon^{-1} U_0$ SAY.

ONLY THEN DO WE GET AN APPARENTLY

ASYMMETRIC COD-2 FBP FOR

A SYMMETRIC IMPACTOR :

$$\int_{d_-}^{d_+} \frac{[s(t) - f(\zeta - Ut)]\zeta^j}{\sqrt{(d_+ - \zeta)(\zeta - d_-)}} \mathrm{d}\zeta = 0 \quad \text{for} \quad j = 0, 1. \tag{4}$$

The physical interpretation of these laws of motion is global conservation of mass ($j = 0$) and x-momentum ($j = 1$) and the condition that $\Psi \sim \mathrm{O}(1/r)$ as $r \to \infty$ guarantees finite spatially-integrated kinetic energy.

A straightforward small-time analysis shows that for bodies that are blunt at the point of impact, the tangential velocity is negligible at sufficiently small times, the velocities of the turnover points being unbounded as $t \downarrow 0$. Hence, the generalized Wagner theory described in this section applies at sufficiently small times. In contrast, for bodies that are linear on each side of the point of impact, the tangential velocity has a leading-order effect no matter how small the time, the velocities of the turnover points being bounded as $t \downarrow 0$. Hence, it is not clear *a priori* whether Wagner theory is applicable at sufficiently small times. We now investigate this case further for the wedge $f(x) = |x|$.

2.1.1. *Grazing entry of a wedge*

Assuming $d_- < Ut < d_+$ and integrating (4), we find

$$\pi s = (2Ut - d_- - d_+) \sin^{-1}\left(\frac{2Ut - d_- - d_+}{d_+ - d_-}\right) + 2\sqrt{Ut(d_- + d_+) - d_- d_+ - U^2 t^2}, \tag{5}$$

$$\begin{aligned} \frac{\pi}{2}(d_- + d_+)s = {} & \left(Ut(d_- + d_+) - \frac{3}{4}(d_-^2 + d_+^2) - \frac{1}{2}d_- d_+\right) \sin^{-1}\left(\frac{2Ut - d_- - d_+}{d_+ - d_-}\right) \\ & + \left(\frac{3}{4}(d_- + d_+) - Ut\right)\sqrt{Ut(d_- + d_+) - d_- d_+ - U^2 t^2}. \end{aligned} \tag{6}$$

AS POINTED OUT BY MOORE (2012)

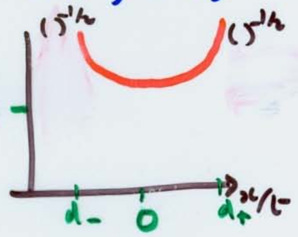
$3/4$ SHOULD BE $3/2 \Rightarrow d_+ = -d_-$

FOR SYMMETRIC IMPACTOR.

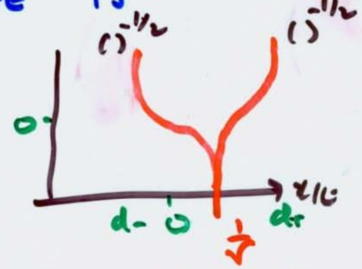
BUT STREAMLINE PATTERN IS <u>NOT</u>

SYMMETRIC AND PRESSURE DISTRIBUTION

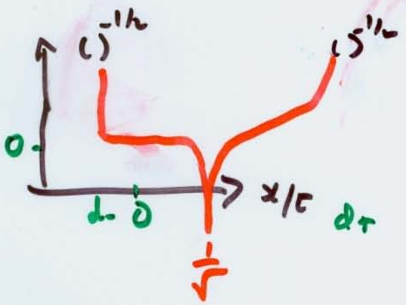
ON, SAY, A WEDGE IS



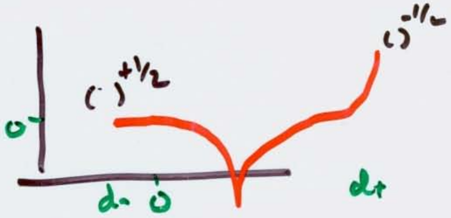
$U_0 = 0$



$U_0 = 0.5$



$U_0 = 1$



$U_0 = \pi/2$

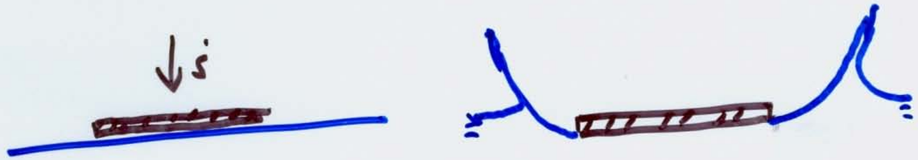
WHEN $U_0 = \pi/2$ WE EFFECTIVELY HAVE

AN EXIT PROBLEM AT d_-.

NON-SMOOTH IMPACTORS

PROBLEM OF IMPACT OF A HORIZONTAL FLAT PLATE WAS FIRST SOLVED BY IAFRATI + KOROBKIN. (P.O.F. 2004)

THEY PROPOSED



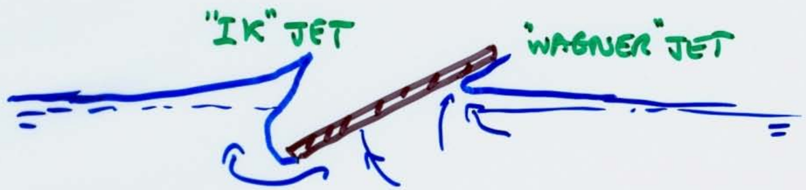
$t = 0$ $\qquad$ $t > 0$

THIS ASSUMES SMOOTH SEPARATION OF FLOW AT ENDS OF PLATE (cf. KUTTA-JOUKOWSKI)

ALSO SPLASH JET IS STILL DESCRIBED BY SHALLOW WATER THEORY BUT NOW $h \sim s^{-5}$ AT ∞.

JET TIP MOVES FROM PLATE TIP TO ∞ IN ZERO TIME !

HOWEVER, FOR AN <u>INCLINED</u> FLAT PLATE, FALTINSEN / SEMENOV HAVE CALCULATED :



ON THIS BASIS OLIVER (2012) HAS CONJECTURED THAT FOR

$$f(x) = x^n , \quad s(t) = t^m$$

IK $n = \infty$, $m = 1$

FS $n = 1$, $m = 1$